

# Managing Project Failure Risk Through Contingent Contracts in Procurement Auctions

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Procurement auctions are sometimes plagued with a chosen supplier's failing to accomplish a project successfully. The risk of project failure is considerable, especially when the buyer has inadequate information about suppliers *ex ante* and the project can only be evaluated at the end. To manage such uncertainty, a model of competitive procurement and contracting for a project is presented in this paper. We study a setting in which suppliers differ in both the costs to fulfill the project and the types reflecting their success probabilities. To screen suppliers, the buyer invites suppliers to specify a two-dimensional bid composed of the proposed cost and a penalty payment if the delivered project fails to meet the requirements. We find that a quasi-linear scoring rule can effectively separate suppliers regarding their types. We then study the efficient and optimal design of the scoring rule. The efficient design internalizes the inferred information on suppliers' type and essentially ranks suppliers based on the expected total cost to the buyer. In the optimal design, the buyer may or may not under-reward suppliers' high success probability, depending on the balance between suppliers' success probabilities and the associated cost distributions. Interestingly, it is always optimal for the buyer to possibly award the project to suppliers with low success probability to promote the competition, even when the difference in suppliers' success probabilities is huge. We show that, compared to standard auctions, the procurement auctions with contingent contracts can significantly improve both social welfare and the buyer's payoff.

*Key words:* procurement auctions; contingent contracts; scoring rules

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## 1. Introduction

Procurement auctions play an important role in eliciting suitable suppliers or contractors for various projects (Beall et al. 2003). In procurement auctions, buyers often spend most of their effort in finding the best contract with the lowest price. However, another critical issue—the ability of a potential supplier to fulfill the project and the associated uncertainty, which are typically unobservable and thus noncontractible—should never be neglected, because it would otherwise lead to disastrous outcomes. In this paper, we propose procurement auctions with contingent contracts and study the design of such auctions to handle this critical issue.

In 1993, the Oregon Department of Motor Vehicles (DMV) launched an information technology (IT) project aiming to computerize its manual and paper-

based daily work. Only two years after its initiation, this initially five-year and \$50 million project was shifted to an estimation of completion in eight years with the cost of \$123 million. In 1996, a prototype of the new system was tested. In less than half a week, the DMV office had its longest waiting line and most complaints ever. The usability of the system was so bad that the system was considered a total failure. Eventually, state officials had no choice but to kill the project completely. Ironically, the IT project that was supposed to downsize the workforce, after tens of millions of dollars of waste, succeeded in nothing but downsizing the DMV officials who witnessed the disaster (Martin 2002).

Large projects (e.g., large-scale IT implementation and military weapon programs) have a certain possibility of failure, partially because of their complexity.

More importantly, the success of a procurement project critically depends on the contractor's project management and quality control capabilities. Without considering the uncertainty of the outcome and hence the possibility of failure, it is impractical for buyers to eliminate the risk by simply specifying all the required outcomes in the contract. Once the project fails, the loss incurred to the buyer is inevitable, and most times it is unbearable. Although buyers could resort to the court for enforced compensation once contractors fail to fulfill the prespecified requirement, the huge operational cost associated with legal procedures and even the possibility of bankruptcy of the contractors make the feasibility and effectiveness of this approach quite limited. Alternatively, buyers may rely on renegotiation to control the loss *ex post*. Nevertheless, as the contractor typically has more information than the buyer regarding the status of the project and the operations being followed, the buyer is often in a disadvantaged position in the renegotiation. Depending on the negotiation power, some projects can take longer than they should and cause enormous cost overrun problems. According to a report from a U.S. Congressional auditing agency in 2008, nearly 70% of the Pentagon's 96 largest weapons programs suffered from huge cost overruns, for a combined total of \$296 billion more than the original estimates (Drew 2009). In many other cases, renegotiation may not even be possible.

To better manage the risk associated with a project, the buyers should take into account the difference among potential suppliers' ability to successfully accomplish the project in the first place, in addition to the cost that the suppliers request. The potential success rate can vary greatly across different contractors, depending on contractors' experience and expertise. On the one hand, the buyer could try to verify each potential contractor's qualification and to assess their expected performance, but such verification and assessment typically involve case-by-case investigation and perhaps on-site involvement, and are thus very costly (Wan and Beil 2009). In some other cases, auctioneers may be able to infer potential contractors' capability by their past performance information (Rothkopf and Whinston 2007), such as the typical approach search engines adopt in online keyword advertising auctions. However, in general,

such information can be difficult or even impossible to collect (e.g., past project history may not be available in sensitive industries like weapons manufacturing). On the other hand, potential suppliers have better knowledge of their own experience, expertise, and capability of managing the project, and thus are able to form a more precise estimation of the success rate. It could be most efficient to induce bidders to self-reveal private information on their success probabilities in procurement auctions. In this sense, it seems extremely appealing to devise a novel mechanism to screen potential contractors so that the most suitable contractor can distinguish itself and win the project as a result. Fortunately, as we show in this paper, by introducing contingent payments into procurement auctions, we are able to achieve this goal.

We study a procurement setting in which both price and nonprice attributes matter. In particular, suppliers differ in both their costs to fulfill the project and their types in terms of the probabilities of their successfully accomplishing the project. Suppliers may have a low or high success probability. Different from price-only auctions or typical scoring auctions, suppliers' types are noncontractible, and thus cannot be incorporated in a scoring function. We propose a contingent contract in determining the winner. Suppliers place bids on fulfillment cost and a penalty payment if they fail to meet the project requirement at the end. The buyers use a scoring rule to determine the winner. We then study the suppliers' equilibrium bidding, investigate the efficient and optimal design of such auctions, and compare the proposed auction approach with standard procurement auctions.

We find that attaching contingency terms with a scoring rule has a significant impact on suppliers' equilibrium bidding. First, a quasi-linear scoring rule can effectively separate suppliers regarding their success probabilities. Suppliers' equilibrium bids on the penalty signal their confidence and reveal their types. Second, suppliers' bids on cost take into account the possible penalty in case of failure and are, not surprisingly, higher than in standard auctions. Furthermore, different scoring rules influence how suppliers with different success probabilities compete with each other in equilibrium and affect the allocation of the project.

Despite the complexity of equilibrium bidding strategies of the bidders, the efficient design is remarkably simple: It internalizes the inferred information on suppliers' types and essentially ranks suppliers based on the expected total cost to the buyer. Relative to the efficient design, the optimal design may or may not under-reward suppliers' high success probabilities, depending on the balance between success probability and cost distribution. Interestingly, to promote the competition between different types of suppliers, the optimal design always ensures that some suppliers with low success probability can win over some of those with high success probability, regardless of the difference of their success probabilities. We show that, compared to standard auctions, the procurement auctions with contingent contracts can significantly improve both social welfare and the buyer's payoff.

The rest of this paper is organized as follows. In §2, we discuss the related literature. We set forth our model in §3. In §4 we explore suppliers' equilibrium bidding behaviors. We examine the design of efficient scoring rules and optimal scoring rules in §5 and §6, respectively. Section 7 discusses some extensions, and §8 concludes this paper.

## 2. Literature Review

Two streams of literature are related to this work. The first stream is the study of scoring rules and scoring auctions. Scoring rules have long been discussed in decision science literature (Kilgour and Gerchak 2004, Johnstone 2007, Bickel 2007, Abbas 2009). Good (1952) was among the first who interpreted scoring rules as incentive devices to induce agents' true belief for assessing a project. Since then, many scoring rules, such as quadratic, spherical, logarithmic, and the parimutuel Kelly scoring rules (Johnstone 2007), have been introduced and investigated for different purposes, such as predicting the probability of an event. Bickel (2007) compares different scoring rules and offers some guidance as to which rule is the most appropriate for which purpose. Unlike the settings studied in the above literature, our paper focuses on procurement auctions using a scoring rule to determine a winner.

This research is closely related to previous studies on "scoring auctions," or auctions in which bidders place multidimensional bids and are ranked by

scores calculated from their bids (Che 1993, Bushnell and Oren 1994, Asker and Cantillon 2008). For example, Che (1993) studies a form of scoring auction used in government procurement, where suppliers bid on both price and quality, and bids are evaluated by a scoring rule. The critical difference of our study from theirs lies in the contingency terms in the auction rule and, fundamentally, the supplier-dependent success rate, which itself cannot be used for scoring rules. In particular, in their setting, bids (on price and nonprice attributes) fully determine the buyer's payoff, whereas in our setting, in addition to their bids, suppliers' success probabilities play an important role in the buyer's payoff.

Scoring rules also appear to be an important component in other auction settings. Bushnell and Oren (1995) consider auctions to select an internal supplier for intermediate products, in which suppliers bid on a fixed cost plus a unit price and the winner is determined by scores calculated from their two-dimensional bids. Liu et al. (2009) study the scoring rule under keyword auctions. They consider contingent payment in auction design under the assumption that the auctioneer can observe the nonprice information, and winner determination is based on both the observed information and the bid. In our case, we assume both the price and nonprice information is private information, and winner determination is according to suppliers' bids on both cost and penalty. In a sense, Liu et al. (2009) take a learning approach, and here we take an economic screening approach.

The second stream of work related to ours is the study of auctioning contingent contracts. Ewerhart and Fieseler (2003) study unit-price-contract auctions in a procurement setting, in which bidders bid unit prices for labor and materials, and the final payment depends on the realized amount of input used. The buyer specifies an estimated amount of each input to evaluate suppliers' bids (using a weighted sum of unit-price bids by the specified amounts), and the bidder with the lowest estimated cost wins. McAfee and McMillan (1986) consider a contractor's moral hazard problem of cost control in procurement auctions and characterize the optimal linear contract. The auction model in this paper differs from the above in the application setting and contract formats, and thus differs in bid structure and equilibrium bidding

behavior. In addition, unlike the equilibrium bidding being determined by a single parameter in the above auctions, equilibrium bidding in our paper is determined both by suppliers' costs and by their success probabilities.

### 3. Model

We consider a risk-neutral buyer seeking to award an indivisible project to a supplier. There is risk associated with the project such that a chosen supplier may not be able to complete the project successfully or meet essential requirements. Examples of such risks include a chosen supplier who cannot deliver the project on time, or the project delivered does not meet the prespecified quality. Therefore, the buyer cares about both the price that a supplier asks and the supplier's type, which reflects its suitability to or preparation for the project. We focus on the setting where the outcome of the project is verifiable so that it is possible to write a contingent contract on the outcome. The buyer hence introduces a contingent contract into the procurement auctions to screen potential suppliers. In particular, the buyer requires bidders to propose a contingent payment when the project fails, in addition to their bids on the cost of completing the project.

There are  $n (\geq 2)$  potential suppliers available in the candidate pool. We assume that suppliers differ in two dimensions: One dimension is the expected cost of completing the project,  $c_i$ . Suppliers may differ in their fulfillment costs because they may use different technologies, human capitals, or procedures. The other dimension is the *type* of supplier, reflecting the probability of its successfully completing the project. We focus on the case where the probability of successful completion for bidder  $i$ ,  $1 - q_i$ , is an inherent characteristic of the bidder; that is,  $q_i$  measures supplier  $i$ 's probability of failing, which characterizes supplier  $i$ 's *type*—its suitability, preparation, or qualification for the project. Throughout this paper, we use the term "type" to refer *exclusively* to suppliers' probability of success (rather than both dimensions). Suppliers may be different in the probability of failure because they may have different levels of expertise, experience, and capability of project management and quality control. In the extension, we discuss the case where suppliers'

types are characterized by distributions of the degree to which they meet the buyer's requirement.

Suppliers privately observe their own cost and type, but not those of the other suppliers. The buyer observes neither suppliers' costs nor their types, so auctions are naturally adopted to choose a contractor. However, the distribution of suppliers' costs and types is common knowledge. Cost  $c$  and probability measure  $q$  are jointly drawn from  $[\underline{c}, \bar{c}] \times [0, 1]$ , following a joint probability density function  $f(c, q)$ . We assume that, for any given  $q_i$ , the density function  $f(c, q_i)$  is positive and continuous on the interval  $[\underline{c}, \bar{c}]$ .

The buyer invites potential suppliers to bid on  $(b_i, t_i)$ , where  $b_i$  is the cost of fulfilling the project and  $t_i$  is the penalty that a supplier is willing to bear if the project delivered fails to meet the essential requirements. We assume suppliers are risk neutral so their payoff functions can be formulated as

$$U(b, t; c, q) = (b - c - qt) \Pr(\text{win}). \quad (1)$$

The buyer derives value  $v$  if the delivered project succeeds in meeting all the requirements. Otherwise, the value is discounted—we assume that the buyer bears loss  $z$  from  $v$ . Therefore, if supplier  $i$  who proposes  $(b_i, t_i)$  wins the contract, the expected payoff for the buyer is

$$V(b_i, t_i, q_i) = v - b_i - (z - t_i)q_i.$$

One important feature of the buyer's payoff differentiates our setting from those of others (e.g., Che 1993, Asker and Cantillon 2008): In our case the buyer not only cares about a supplier's bid (the same as in other settings), but is also concerned about the supplier's true type, reflecting its probability of success (which itself, as a probability measure, is unobservable and noncontractible, and thus cannot be made part of an agreement).

The project is assigned according to some scoring rule  $S(b, t)$  preannounced by the buyer. The scoring function is increasing in  $b$  and decreasing in  $t$ . The supplier with the lowest score wins the project. In particular, we consider a quasi-linear scoring function in the form of  $S(b, t) = b - \Lambda(t)$ , where  $\Lambda(t)$  is increasing and concave (i.e.,  $\Lambda'(t) > 0$  and  $\Lambda''(t) < 0$ ). In other words, we consider a class of scoring rule in which bids on price and bids on penalty are separable and additive. This class of scoring rule encompasses many commonly used scoring rules. For example,

$\Lambda(t) = w\sqrt{t}$  represents the simplest square-root scoring rule, and  $\Lambda(t) = w \ln(1+t)$  represents a logarithmic scoring rule.

Next, we start with a simple case of two possible types:  $q_l$  and  $q_h$ ,  $0 < q_l < q_h < 1$ . We call the suppliers with  $q_l$  the *preferred* suppliers and the suppliers with  $q_h$  the *nonpreferred* suppliers, because the former has a lower failure probability and, everything else being equal, is more desirable to the buyer. We will extend our model to a multiple-type case in §7. We assume that the probabilities for an individual supplier's being preferred and being nonpreferred are  $\alpha$  and  $1-\alpha$ , respectively. To simplify the notation, we let  $f_l(c) \equiv f(c, q_l)$  and  $f_h(c) \equiv f(c, q_h)$ , which have a fixed and common support  $[\underline{c}, \bar{c}]$ . Correspondingly, we define the cumulative distribution functions  $F_l(c) = \int_{\underline{c}}^c f_l(x) dx$  and  $F_h(c) = \int_{\underline{c}}^c f_h(x) dx$ . The common support assumption is to simplify notation. The analysis and results can be easily generalized to the cases with different supports  $[\underline{c}_l, \bar{c}_l]$  and  $[\underline{c}_h, \bar{c}_h]$  for the preferred and the nonpreferred suppliers, respectively.

To avoid distraction from trivial equilibrium outcomes, throughout this paper, we focus our discussion on the case in which the difference in suppliers' success probabilities is not too huge so that in equilibrium at least some low-cost nonpreferred suppliers can win over some high-cost preferred suppliers. In other words, we focus our discussion on the scoring rules that satisfy

$$[\Lambda(\Lambda'^{-1}(q_l)) - q_l \Lambda'^{-1}(q_l)] - [\Lambda(\Lambda'^{-1}(q_h)) - q_h \Lambda'^{-1}(q_h)] < \bar{c} - \underline{c}. \quad (2)$$

The explanation of this condition shall become clear after Proposition 1. When condition (2) is not satisfied, the equilibrium outcome reduces to a simple case in which suppliers only compete with peers of the same type, and similar analysis remains to apply.

#### 4. Equilibrium Analysis

We first consider suppliers' bidding strategies. Throughout this paper, we consider a symmetric, pure-strategy Bayesian-Nash equilibrium. By "symmetric" we mean that suppliers with the same cost and type will bid the same.

Let  $\tau \equiv b - c - tq$ , which is the expected profit a supplier earns upon winning the auction. Given any

such supposed profit level, the supplier intends to maximize the winning probability or, equivalently, to minimize its score by properly choosing the bid on penalty  $t$ :

$$\min_t S(\tau + c + qt, t) = \tau + c + qt - \Lambda(t). \quad (3)$$

Because of the concavity of  $\Lambda(\cdot)$ , the first-order condition characterizes the optimal bid on the penalty:  $q = \Lambda'(t^*)$ . For example, for the class of logarithmic scoring rule with  $\Lambda(t) = w \ln(1+t)$ , it is easy to derive  $t^* = 1/q - 1$ ; for the class of square-root scoring rule with  $\Lambda(t) = w\sqrt{t}$ ,

$$t^* = \frac{w^2}{4q^2}. \quad (4)$$

We denote the optimal bid on penalty as

$$t^*(q) \equiv \Lambda'^{-1}(q). \quad (5)$$

**LEMMA 1.** *Under the scoring rule  $S(b, t)$ , in equilibrium the suppliers bid  $t^*(q)$  on the penalty part as defined by Equation (5). Moreover,  $t^*(q)$  is decreasing in  $q$ .*

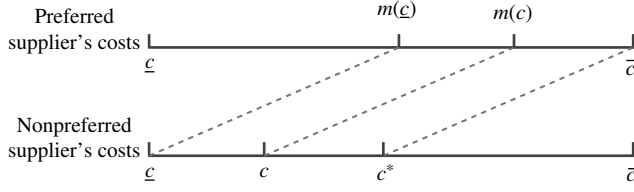
**PROOF.** Because  $\Lambda''(t) < 0$ ,  $\Lambda'^{-1}(q)$  is well defined and decreasing in  $q$ . Therefore,  $t^*(q)$  is decreasing in  $q$ .  $\square$

The above lemma indicates that in equilibrium bidders with lower failure probability bid more on penalty. In this sense, the quasi-linear scoring function can effectively screen bidders in terms of their private types: The more suitable or better prepared suppliers bid more aggressively on penalty.

We next examine suppliers' bidding strategies on the cost. Based on the above lemma, we now rewrite the suppliers' payoff function Equation (1) as

$$\begin{aligned} U(b, t^*(q_h); c, q_h) &= (b - c - q_h t^*(q_h)) \Pr(\text{win}), \\ U(b, t^*(q_l); c, q_l) &= (b - c - q_l t^*(q_l)) \Pr(\text{win}). \end{aligned} \quad (6)$$

We denote  $b_l(c)$  and  $b_h(c)$  as the equilibrium bids on the fulfilling cost for suppliers with  $q_l$  and  $q_h$ , respectively. We conjecture and will verify that both bidding functions are increasing in supplier's cost  $c$ , and there exists  $c^* \in [\underline{c}, \bar{c}]$  such that  $S(b_l(\bar{c}), t^*(q_l)) = S(b_h(c^*), t^*(q_h))$ . The conjecture on  $c^*$  is intuitive because, in general, suppliers with a lower failure rate have a greater chance to win the contract, and a preferred supplier with the highest possible cost may

**Figure 1** Comparable Bidders

have the same winning probability as some non-preferred supplier with a lower cost. Under this conjecture, for any non-preferred supplier with cost  $c$  below  $c^*$ , there must be a preferred supplier with cost  $m(c)$  who matches the former's score, where  $m: [\underline{c}, \bar{c}] \mapsto [\underline{c}, \bar{c}]$  is a mapping from a non-preferred supplier to its counterpart in the preferred group. We call this pair of suppliers *comparable* bidders. Figure 1 illustrates a pair of comparable bidders by a dashed line, with a non-preferred supplier of cost  $c$  at one end and a preferred supplier of cost  $m(c)$  at the other. Technically, we simply define  $m(c) = \bar{c}$  for  $c > c^*$ , and  $m^{-1}(c) = \underline{c}$  for  $c < m(\underline{c})$ . Based on such notations, we can formulate the suppliers' equilibrium winning probabilities:

$$\rho_l(c) = [\alpha(1 - F_l(c)) + (1 - \alpha)(1 - F_l(m^{-1}(c)))]^{n-1},$$

$$\rho_h(c) = [\alpha(1 - F_l(m(c))) + (1 - \alpha)(1 - F_l(c))]^{n-1}.$$

A non-preferred supplier with cost  $c$  can beat a competitor only if the competitor is a non-preferred supplier with a higher cost, or if the competitor is a preferred supplier but with a cost higher than  $m(c)$ . The non-preferred supplier can win the contract only if it beats all the remaining  $n - 1$  competitors, which explains  $\rho_h(c)$ . The winning probability for the preferred suppliers  $\rho_l(c)$  is derived along a similar line.

**PROPOSITION 1.** *Under the scoring rule  $S(b, t)$ , in equilibrium, suppliers place bids on penalty as in Lemma 1 and place bids on cost as follows.*

$$b_h(c) = c + q_h t^*(q_h) + \frac{\int_c^{\bar{c}} \rho_h(x) dx}{\rho_h(c)},$$

$$b_l(c) = c + q_l t^*(q_l) + \frac{\int_{c^*}^{\bar{c}} \rho_h(x) dx + \int_c^{c^*} \rho_l(x) dx}{\rho_l(c)},$$
(7)

where  $m(c) = c + \Delta$ ,  $c^* = \bar{c} - \Delta$ , and  $\Delta$  is defined by

$$\Delta \equiv [\Lambda(t^*(q_l)) - q_l t^*(q_l)] - [\Lambda(t^*(q_h)) - q_h t^*(q_h)]. \quad (8)$$

**PROOF.** All proofs of propositions are deferred to the appendix.  $\square$

We can easily verify that the above bidding functions are indeed monotonically increasing in cost  $c$  by checking the first-order derivative. As in the standard auctions, in equilibrium suppliers ask more than their true cost.

Two features of the bidding functions differ from that in the standard auctions. First, in the bid on cost, we have an extra term of  $q t^*(q)$ , which is exactly the expected penalty. Once suppliers promise to bear the penalty if they fail to meet the requirement, they take such possible revenue loss into account in formulating their bidding. So the buyer is not better off from the suppliers' promised penalty itself. Instead, the equilibrium penalty serves like a screening device that separates their suppliers with different types. Second, in the preferred suppliers' bidding function, we observe one extra term in the fraction  $\int_{c^*}^{\bar{c}} \rho_h(x) dx$ , compared to that of a standard auction. We call it the *base payoff* for preferred suppliers. Unlike in the standard auctions, in equilibrium, the preferred supplier with even the highest possible cost  $\bar{c}$  can still earn a certain level of payoff, because it has a lower failure rate and is more desirable, from the buyer's perspective, than some non-preferred suppliers with lower cost.

The revealed matching pattern  $m(c) = c + \Delta$  indicates that a non-preferred supplier with cost  $c$  and a preferred supplier with cost  $c + \Delta$  are comparable (i.e., they have the same winning probability). Intuitively, because  $(b - qt)$  represents a supplier's expected revenue from the buyer,  $\Delta$  defined in Equation (8) can be interpreted as the difference between a preferred supplier's expected revenue and a non-preferred supplier's expected revenue when they both bid the same score  $s$  (because  $\Delta = [s + \Lambda(t^*(q_l)) - q_l t^*(q_l)] - [s + \Lambda(t^*(q_h)) - q_h t^*(q_h)]$ ). A preferred supplier of cost  $c + \Delta$  and a non-preferred supplier of cost  $c$  thus have the same expected profit (i.e., expected revenue less fulfillment cost) upon winning, if they bid the same score. Therefore, the two suppliers face the same trade-off between the expected profit upon winning and probability of winning, which leads to the same choice of score in equilibrium. Technically, when a non-preferred supplier with cost  $c$  bids score  $s'$  (i.e., bid  $s' + \Lambda(t^*(q_h))$  on cost) and a preferred supplier

with cost  $c + \Delta$  bids score  $s''$  (i.e., bid  $s'' + \Lambda(t^*(q_l))$  on cost), we can rewrite their payoffs as

$$\begin{aligned}
 & U(s' + \Lambda(t^*(q_h)), t^*(q_h); c, q_h) \\
 &= (s' + \Lambda(t^*(q_h)) - c - q_h t^*(q_h)) \\
 &\quad \cdot \Pr(s' \text{ is the lowest score}), \\
 & U(s'' + \Lambda(t^*(q_l)), t^*(q_l); c + \Delta, q_l) \\
 &= (s'' + \Lambda(t^*(q_l)) - c - \Delta - q_l t^*(q_l)) \\
 &\quad \cdot \Pr(s'' \text{ is the lowest score}) \\
 &= (s'' + \Lambda(t^*(q_h)) - c - q_h t^*(q_h)) \\
 &\quad \cdot \Pr(s'' \text{ is the lowest score}),
 \end{aligned}$$

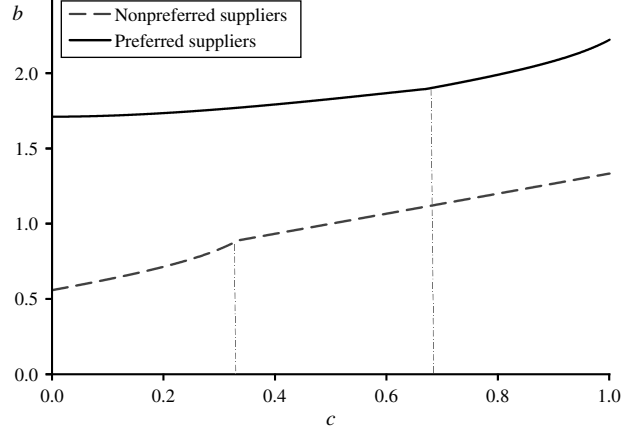
where the last equality is achieved by substituting  $\Delta$  defined in Equation (8). Evidently, those two suppliers face exactly the same optimization problem when choosing the optimal score. That is, if  $s^*$  is the best choice for the preferred supplier with  $c + \Delta$ ,  $s^*$  must be the best choice for the nonpreferred supplier with cost  $c$  as well. Therefore, the nonpreferred supplier with cost  $c$  has the same winning probability as the preferred supplier with cost  $c + \Delta$ ; or those two are comparable in equilibrium.

Notice that the term on the left-hand side of Equation (2) is simply the definition of  $\Delta$  in Equation (8). So condition (2) is to ensure that  $\Delta < \bar{c} - \underline{c}$ . When the condition is not satisfied (i.e.,  $\Delta \geq \bar{c} - \underline{c}$ ), in equilibrium, the preferred suppliers adjust their bids so that the preferred supplier with the highest cost  $\bar{c}$  ties in score with the nonpreferred one with the lowest cost  $\underline{c}$ . This is because, for the preferred supplier with the highest cost, bidding the same score as the nonpreferred one with the lowest cost is the most profitable way to win over all realized nonpreferred suppliers. As a result, none of the nonpreferred suppliers can win over any preferred supplier. More explicitly,  $b_h(c)$  takes the same form as in Equation (7), and<sup>1</sup>

$$\begin{aligned}
 b_l(c) &= c + q_l t^*(q_l) \\
 &+ \frac{\int_c^{\bar{c}} \rho_l(x) dx + \int_{\underline{c}}^{\bar{c}} \rho_h(x) dx + (\Delta - \bar{c} + \underline{c}) \rho_l(\bar{c})}{\rho_l(c)}, \quad (9)
 \end{aligned}$$

<sup>1</sup>  $b_h(c)$  and  $b_l(c)$  are derived in a manner similar to that for Proposition 1 except that  $U(b_l(\bar{c}), t^*(q_l); \bar{c}, q_l)$  is determined by  $b_l(\bar{c}) - \Lambda(t^*(q_l)) = b_h(\underline{c}) - \Lambda(t^*(q_h))$  instead.

Figure 2 Example of Equilibrium Bidding Strategies



where  $\rho_l(c) = [\alpha(1 - F_l(c)) + (1 - \alpha)]^{n-1}$  and  $\rho_h(c) = [(1 - \alpha) \cdot (1 - F_h(c))]^{n-1}$ .

The following example illustrates suppliers' equilibrium bidding.

EXAMPLE 1. We consider a setting in which  $\alpha = 1/2$ ,  $q_l = 1/4$ ,  $q_h = 3/4$ , and  $n = 3$ . We assume the costs of the preferred and the nonpreferred suppliers are distributed over  $[0, 1]$ , with the cumulative distribution functions  $F_l(x) = x^2$  and  $F_h(x) = x$ , respectively. We use the square-root scoring rule with  $w = 1$  (i.e.,  $\Lambda(t) = \sqrt{t}$ ) as an example. According to Equation (4), equilibrium bids on penalty are  $t^*(q_l) = 4$  and  $t^*(q_h) = 4/9$ . Clearly, the preferred suppliers bid more than the nonpreferred suppliers on penalty. According to Equation (8), we can calculate  $\Delta = 2/3$ , and thus  $v^* = 1/3$  and  $m(c) = c + 2/3$ . We plot suppliers' equilibrium bids on cost as in Figure 2.

It is worth highlighting the kink in each bidding function. In Figure 2, we observe a kink at  $2/3$  in the preferred suppliers' bidding function and a kink at  $1/3$  in the nonpreferred suppliers' bidding function. Intuitively, the presence of such a kink is because of the change in competition suppliers face. For example, preferred suppliers with low cost have no comparable nonpreferred suppliers, and thus the competition is mainly within the preferred group. Once preferred suppliers' cost reaches a certain level ( $c = 2/3$  in the above example), they face the competition from the comparable nonpreferred suppliers, in addition to their own peers, which changes the competition they face and results in the kink in their bidding function.

A similar explanation applies to the kink in the non-preferred suppliers' bidding function.

## 5. Efficiency and Scoring Rule

In this section, we examine the efficiency of different scoring rules. We measure the efficiency by the total value created. The efficiency criterion, therefore, emphasizes the "total pie," which is especially important for procurement in the public sector (i.e., government procurement).

Because of the monotonicity in the equilibrium bidding on cost analyzed in the previous section, for any given bidders with realized costs drawn from the cost distributions, there are only two winning candidates: the preferred bidder with the lowest cost among its peers,  $c_l$ , and the nonpreferred bidder with the lowest cost among its peers,  $c_h$ . The former generates a total value of  $v - q_l z - c_l$ , and the latter generates  $v - q_h z - c_h$  upon being assigned the project. Therefore, an efficient allocation should ensure that the contract be assigned to the preferred lowest-cost supplier if and only if  $v - q_l z - c_l > v - q_h z - c_h$  or, equivalently,  $c_l - c_h < z(q_h - q_l)$ . As is discussed above, in equilibrium, the preferred bidder with cost  $c_l$  wins over its nonpreferred counterpart with cost  $c_h$  if and only if  $c_l < c_h + \Delta$ . Therefore, as long as  $\Delta = z(q_h - q_l)$ , we can ensure efficient allocation no matter what the realized cost distribution among all bidders looks like. We thus show that the scoring rule satisfying the condition  $\Delta = z(q_h - q_l)$  is ex post efficient.

In addition to ex post efficiency, naturally, we are also interested in the scoring rule achieving ex ante efficiency. We say a scoring rule is ex ante socially efficient if it maximizes the *expected* total value generated in equilibrium. Given that the probabilities of assigning the project to preferred and nonpreferred suppliers are  $\rho_l(c)$  and  $\rho_h(c)$ , respectively, the expected total value generated in equilibrium is

$$n\alpha \int_{\underline{c}}^{\bar{c}} (v - q_l z - c) \rho_l(c) f_l(c) dc + n(1 - \alpha) \int_{\underline{c}}^{\bar{c}} (v - q_h z - c) \rho_h(c) f_h(c) dc. \quad (10)$$

**PROPOSITION 2.** *Any scoring rules satisfying  $\Delta = z(q_h - q_l)$  are both ex ante and ex post socially efficient.<sup>2</sup>*

<sup>2</sup>Notice that  $\Delta^{\text{eff}} < \bar{c} - \underline{c}$  requires that  $z(q_h - q_l) < \bar{c} - \underline{c}$ . When  $z(q_h - q_l) \geq \bar{c} - \underline{c}$ , any scoring rule with  $\Delta \geq \bar{c} - \underline{c}$ , under which none

Recall that we define  $\Delta$  in Equation (8). Under the efficient scoring rule, we have

$$\Lambda(t^*(q_l)) - (t^*(q_l) - z)q_l = \Lambda(t^*(q_h)) - (t^*(q_h) - z)q_h.$$

If we denote the value of the above as a constant  $\psi$ , then  $\Lambda(t^*(q_l)) = (t^*(q_l) - z)q_l + \psi$  and  $\Lambda(t^*(q_h)) = (t^*(q_h) - z)q_h + \psi$ . So the winner is essentially determined by

$$b - \Lambda(t^*(q_j)) = b + (z - t^*(q_j))q_j - \psi, \quad j \in \{l, h\}.$$

Notice that  $b + (z - t^*(q_j))q_j$  is just the expected total cost to the buyer. So the efficient scoring rule essentially internalizes the inferred information regarding suppliers' success probabilities and implements a rule that ranks suppliers by their total expected cost to the buyers. Such a scoring rule is efficient because the bidders are "fairly" evaluated in the sense that the winner is the one who can minimize the buyer's expected total cost or maximize the buyer's expected payoff.

It is worth noting that the policy prescribed above is independent of the distribution of suppliers' costs and types. This feature makes it easy to implement an efficient scoring rule: The buyer only needs to estimate the failure rates of suppliers and combine them with its own loss from a failure. In fact, the coefficient  $\Delta$  is exactly the difference in expected loss from different suppliers.

Notice that because we do not specify the form of  $\Lambda(t)$ , many scoring rules with properly set parameters can be efficient. For example, for the class of square-root scoring rule with  $\Lambda(t) = w\sqrt{t}$ , we have  $t^*(q) = w^2/(4q^2)$  by Equation (4) and  $\Delta$  defined in Equation (8) becomes

$$\Delta = \left[ \frac{w^2}{2q_l} - \frac{w^2}{4q_l} \right] - \left[ \frac{w^2}{2q_h} - \frac{w^2}{4q_h} \right] = \frac{w^2}{4q_l} - \frac{w^2}{4q_h}. \quad (11)$$

Because the efficient scoring rule requires  $\Delta = z(q_h - q_l)$ , the efficient coefficient is

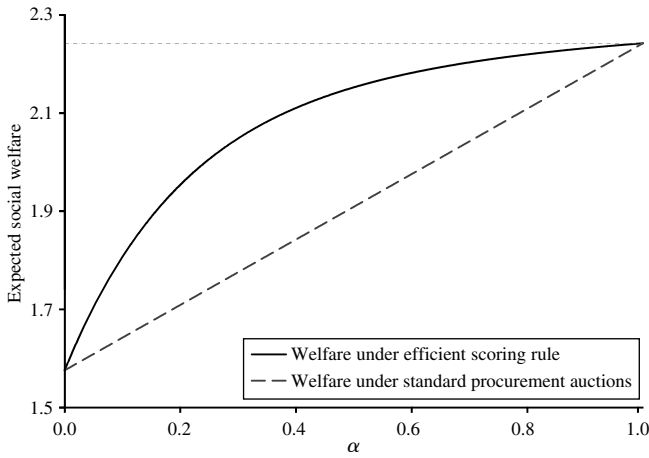
$$w = 2\sqrt{zq_h q_l}. \quad (12)$$

For the logarithmic scoring rule with  $\Lambda(t) = w \ln(1 + t)$ ,  $t^* = 1/q - 1$ , and thus the efficient coefficient  $w$  is characterized by  $-q_h + w \ln q_h + q_l - w \ln q_l = z(q_h - q_l)$ .

of the nonpreferred suppliers wins over the preferred ones in equilibrium (see the discussion around Equation (9)), and thus  $\rho_l(c)$  and  $\rho_h(c)$  are independent of  $\Delta$ , achieves the efficiency.



**Figure 3** Example of Expected Social Welfare



The following example compares the expected social welfare under our auctions with the contingent payments to that under standard procurement auctions. As we can see, the auction format we propose can significantly improve the social welfare, especially when there is considerable uncertainty about the outcome (i.e.,  $\alpha$  is around the middle).

**EXAMPLE 2.** We let  $q_l = 1/3$ ,  $q_h = 2/3$ ,  $z = 2$ ,  $v = 3$ , and  $n = 10$ , and assume the costs of preferred and nonpreferred suppliers are uniformly distributed on  $[0, 1]$ . We use the square-root scoring rule (i.e.,  $\Lambda(t) = w\sqrt{t}$ ) as an example. By Equation (12), the efficient coefficient  $w$  is  $4/3$ . Figure 3 depicts how the expected social welfare achieved with the efficient scoring function changes with  $\alpha$ . We use the expected social welfare achieved in standard procurement auctions (corresponding to  $\Delta = 0$  in our setting) as a benchmark for comparison.

## 6. Optimal Scoring Rule

In addition to the efficiency criterion, another useful design criterion is payoff maximization. Especially in the private sector, firms are typically interested in minimizing their procurement cost. When some nonprice attributes also matter, as in our setting, firms often intend to maximize their expected payoff or minimize the expected total cost (including the possible loss from failure). Next, we examine how a buyer should choose the scoring rule to maximize its own expected payoff.

We can explicitly derive the buyer's expected payoff (see the appendix for details) as

$$v - n\alpha \left[ \int_{\underline{c}}^{\bar{c}} \left[ q_l z + c + \frac{F_l(c)}{f_l(c)} \right] \rho_l(c) f_l(c) dc + \int_{c^*}^{\bar{c}} \rho_h(c) dc \right] - n(1 - \alpha) \int_{\underline{c}}^{\bar{c}} \left[ q_h z + c + \frac{F_h(c)}{f_h(c)} \right] \rho_h(c) f_h(c) dc. \quad (13)$$

The second term is the expected cost from a preferred supplier and the third term is the expected cost from a nonpreferred supplier. In traditional procurement auction settings (with suppliers being the same regarding their type), the revenue is typically in the form of  $v - n \left[ \int_{\underline{c}}^{\bar{c}} [c + F(c)/f(c)] \rho(c) f(c) dc \right]$ , in which  $c + F(c)/f(c)$  is usually called the *virtual cost*. Along a similar line, we can call  $q_l z + c + F_l(c)/f_l(c)$  and  $q_h z + c + F_h(c)/f_h(c)$  virtual costs from a preferred supplier and nonpreferred supplier, respectively. Virtual cost essentially measures each supplier's marginal contribution to the buyer's expected payment. The virtual cost is different from actual cost by a term that captures the externality imposed to other suppliers. Three features in our expected payoff stand out from traditional procurement auctions. First, in addition to their costs, suppliers differ in their types in terms of success probability, and thus we have two terms (one for each type). Second, unlike in the traditional procurement auction setting, we have  $q_l z$  and  $q_h z$  in the virtual cost. This is because suppliers may fail to meet the requirement, and the buyer thus incurs the expected loss from its value  $v$ . Third, because preferred suppliers naturally have an advantage, their base payoff also plays a role in the buyer's expected payoff, as we can anticipate.

We call a scoring rule that maximizes the expected payoff in Equation (13) an optimal scoring rule. The optimal scoring rule can be obtained from the first-order condition of the expected payoff with respect to  $\Delta$ . Except for some special cases, the optimal  $\Delta$  cannot be expressed in an explicit form. Next, we focus on two issues regarding the optimal scoring rule. First, how is it different from the efficient design? Second, how is it affected by the underlying model primitives, especially cost distributions? We introduce the following definition for the purpose of comparison.

**DEFINITION 1.** For two cumulative distribution functions  $\tilde{F}(c)$  and  $F(c)$  (with the density functions

being  $\tilde{f}(c)$  and  $f(c)$ , respectively), we say  $\tilde{F}(c)$  is a distributional upgrade of  $F(c)$  if

$$\frac{\tilde{f}(c)}{\tilde{F}(c)} \leq \frac{f(c)}{F(c)} \quad (14)$$

for all  $c$  where the above expressions are well defined.

The requirement (14) is one of conditional stochastic dominance: Conditional on any maximum level of cost,  $\tilde{F}(c)$  is less likely to yield a higher cost than  $F(c)$ . Similar conditions have often been used in the auction literature. For example, Maskin and Riley (2000) use such a condition when they study asymmetric auctions. Many classes of distributions or changes in distribution can satisfy the condition of distributional upgrade. For instance, shifts of distribution to the left generate a distributional upgrade. By simple algebra, we can verify that a uniform distribution on  $[1, 3]$  is a distributional upgrade of a uniform distribution on  $[2, 4]$ .

**DEFINITION 2.** A distribution  $F(c)$  satisfies the regularity condition if  $F(c)/f(c)$  is increasing in  $c$  for all  $c$  where the expression is well defined.

The regularity condition is commonly adopted in many procurement studies (e.g., Che 1993). Saying that a distribution satisfies a regularity condition is equivalent to saying that the distribution  $F(c)$  is log-concave. Many distributions, including uniform, normal, exponential, and logistic distributions, satisfy the regularity condition.

**PROPOSITION 3.** *The optimal scoring rule is characterized by (first-order condition)*

$$\int_{c+\Delta}^{\bar{c}} \left[ q_h z - q_l z - \Delta + \frac{F_h(c-\Delta)}{f_h(c-\Delta)} - \frac{F_l(c)}{f_l(c)} \right] \frac{d\rho_l(c)}{d\Delta} f_l(c) dc = \rho_h(\bar{c} - \Delta) \quad (15)$$

or by  $\Delta = 0$  (corner solution). If  $F_h(c)$  satisfies the regularity condition and  $F_l(c)$  is a distributional upgrade of  $F_h(c)$ , then the optimal scoring rule satisfies  $\Delta^{\text{opt}} < \Delta^{\text{eff}}$ .

Recall that a nonpreferred supplier with cost  $c$  is comparable to a preferred supplier with  $c + \Delta$  in equilibrium. A lower  $\Delta$  means a greater chance for the nonpreferred supplier to win over a preferred supplier, and thus a higher equilibrium probability of winning. Under the regularity and distributional upgrade conditions, the above proposition suggests

that scoring rules should be biased to promote the nonpreferred suppliers (compared to the “fairness” of the efficient design). In other words, in equilibrium, the buyer under-rewards suppliers with higher success probability for the sake of maximizing its expected payoff.

The bias or under-rewarding can be seen even more clearly from the scoring rule. For example, under the square-root scoring rule  $\Lambda(t) = w\sqrt{t}$ , by the  $\Delta$  derived in Equation (11),  $\Delta^{\text{opt}} < \Delta^{\text{eff}}$  is equivalent to

$$\frac{(w^{\text{opt}})^2}{4q_l} - \frac{(w^{\text{opt}})^2}{4q_h} < \frac{(w^{\text{eff}})^2}{4q_l} - \frac{(w^{\text{eff}})^2}{4q_h},$$

which leads to that  $w^{\text{opt}} < w^{\text{eff}}$ . Recall that preferred suppliers are willing to bear a high penalty  $t$  for project failure in general (by Lemma 1). Reducing  $w$  (from  $w^{\text{eff}}$  to  $w^{\text{opt}}$ ) simply cuts preferred suppliers’ advantage in the scoring rule, and essentially under-rewards preferred suppliers’ advantage of their low failure rate.

The intuition is as follows. First, for any scoring rule with  $\Delta \geq \Delta^{\text{eff}}$ , under the regularity and distributional upgrade conditions, we have

$$\left( q_l z + c + \frac{F_l(c)}{f_l(c)} \right) > \left( q_h z + c - \Delta + \frac{F_h(c-\Delta)}{f_h(c-\Delta)} \right).$$

This is because  $q_l z + c = q_h z + c - \Delta^{\text{eff}}$  and  $F_l(c)/f_l(c) \geq F_h(c)/f_h(c) > F_h(c-\Delta)/f_h(c-\Delta)$ . In other words, under any scoring rule with  $\Delta \geq \Delta^{\text{eff}}$ , the virtual cost for a preferred supplier to fulfill the project is always higher than that of a comparable nonpreferred supplier. Notice that in Equation (13), in addition to the preferred suppliers’ base payoff, the total expected payment is the expectation of suppliers’ weighted virtual costs (with the weights being their respective winning probability). Thus, the buyer obtains higher expected payoff by choosing any scoring rule with  $\Delta < \Delta^{\text{eff}}$ . Second, a scoring rule also affects the preferred suppliers’ base payoff  $\int_{\bar{c}-\Delta}^{\bar{c}} \rho_h(c) dc$ . In particular, the base payoff is increasing in  $\Delta$ . Because the buyer hopes to minimize the preferred suppliers’ base payoff, a scoring rule with  $\Delta < \Delta^{\text{eff}}$  is more desirable than the one with a higher  $\Delta$ . Therefore, the optimal scoring rule must satisfy  $\Delta^{\text{opt}} < \Delta^{\text{eff}}$  under the regularity and distributional upgrade conditions.

It is worth pointing out that the optimal scoring rule prescribed by Proposition 3 is optimal among all

possible  $\Delta$  (including  $\Delta \geq \bar{c} - \underline{c}$ ) and the optimal scoring rule is always less than  $\bar{c} - \underline{c}$ . In other words, it is always beneficial for the buyer to use the scoring rule to leverage the competition between the two groups of suppliers by letting some low-cost nonpreferred suppliers win over some high-cost preferred ones in equilibrium. Intuitively,  $\Delta > \bar{c} - \underline{c}$  cannot be optimal because preferred suppliers' base payoff is increasing in  $\Delta$  (see Equation (9)), and thus a larger  $\Delta$  means a higher base payoff for preferred suppliers and a lower expected payoff for the buyer. The finding is interesting in that even when the difference between the two types of suppliers' success probabilities is so huge that choosing any nonpreferred supplier is apparently inefficient, the optimal choice for the buyer is still to possibly award the project to a nonpreferred supplier to promote the competition between the two groups.

Notice that the distributional upgrade we defined is in a weak form that allows identical distributions. Therefore, for the above proposition, one special case is that suppliers' types are independent of their costs, such that  $F_h(c) = F_l(c)$ . With this independence, the only condition needed is the regularity one.

It is also worth noting that the properties of the optimal scoring rule derived in Proposition 3 (i.e., the value of  $\Delta^{\text{opt}}$  and  $\Delta^{\text{opt}} < \Delta^{\text{eff}}$  under certain conditions) do not depend on the form of  $\Lambda(t)$ , because we do not specify the form of the scoring rule in the analysis. The optimal  $\Delta$  can be calculated from Equation (15), which, in general, is a function of the number of suppliers. For any given class of scoring rule (e.g., the square-root scoring rule  $\Lambda(t) = w\sqrt{t}$ ), the optimal scoring rule (e.g., the optimal  $w$  in the square-root scoring rule) can be computed by substituting the optimal  $\Delta$  into Equation (8). We next use an example to illustrate the optimal scoring rule.

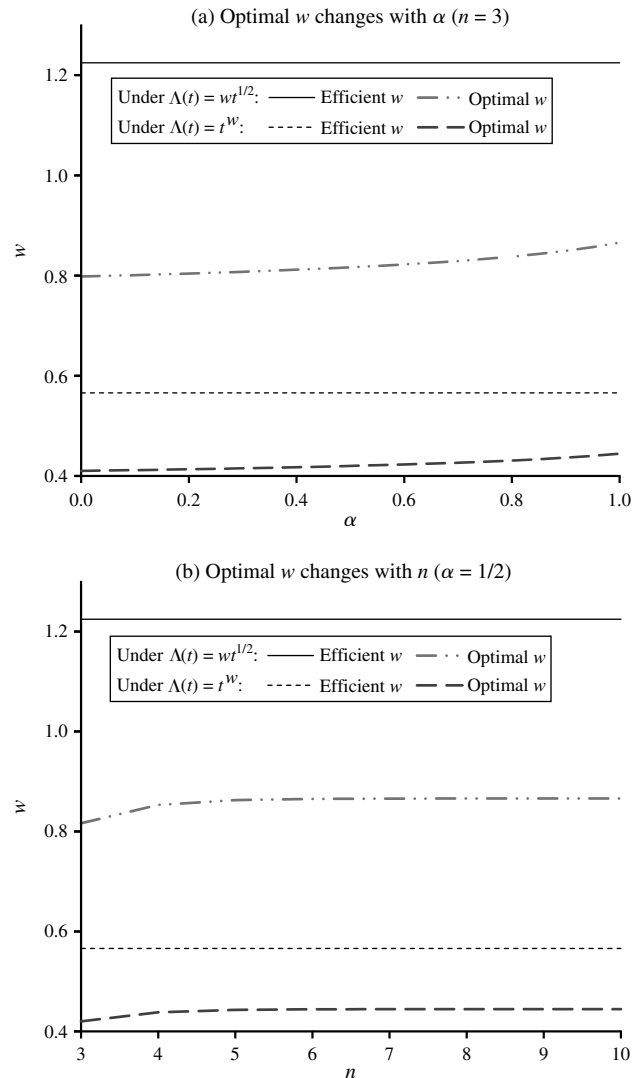
**EXAMPLE 3.** We consider a setting in which  $q_l = 1/4$ ,  $q_h = 1/2$ ,  $z = 3$ , and  $v = 3$ . Assume the costs of preferred and nonpreferred suppliers are uniformly distributed on  $[0, 1]$ . Given  $\alpha$  and  $n$ ,  $\Delta^{\text{opt}}$  can be derived by the first-order condition characterized by Equation (15). For example, when  $\alpha = 1/2$  and  $n = 3$ , we can obtain  $\Delta^{\text{opt}} = 1/3$ , which is less than  $\Delta^{\text{eff}} = z(q_h - q_l) = 3/4$ . Furthermore, we can check the optimal parameters of different scoring rules. For instance, under the square-root scoring rule, by substituting  $\Delta^{\text{opt}}$  and  $\Delta^{\text{eff}}$  into Equation (11), we can

derive  $w^{\text{opt}} = \sqrt{2/3}$  and  $w^{\text{eff}} = \sqrt{3/2}$ , respectively. Under the scoring rule with  $\Lambda(t) = t^w$  ( $0 < w \leq 1$ ),  $\Delta$  can be formulated as

$$\Delta = \left[ \left( \frac{q_l}{w} \right)^{w/(w-1)} - q_l \left( \frac{q_l}{w} \right)^{1/(w-1)} \right] - \left[ \left( \frac{q_h}{w} \right)^{w/(w-1)} - q_h \left( \frac{q_h}{w} \right)^{1/(w-1)} \right]. \quad (16)$$

We can thus derive that  $w^{\text{opt}} \simeq 0.42$  and  $w^{\text{eff}} \simeq 0.57$ . Clearly,  $w^{\text{opt}} < w^{\text{eff}}$  in both cases, which implies that under the optimal weighting scheme the buyer understates the importance of the penalty to create competition pressure for the preferred suppliers. Figure 4

**Figure 4** Example of Optimal  $w$



presents how the optimal parameter  $w^{\text{opt}}$  changes with  $\alpha$  and with the number of bidders  $n$  under different scoring rules. We see that  $w^{\text{opt}}$  increases in  $n$  and in  $\alpha$ , because the competition within preferred suppliers increases in  $n$  and in  $\alpha$ , reducing the need to induce competition by promoting nonpreferred suppliers. In addition, by substituting  $\Delta^{\text{opt}}$  into Equation (13), we can calculate the maximum expected payoff under the optimal scoring rule, 1.46, which is higher than that under standard procurement auctions, 1.38.

When the cost distributions for the preferred and nonpreferred suppliers do not satisfy the conditions in Proposition 3, however, the optimal scoring rule may not have  $\Delta^{\text{opt}} < \Delta^{\text{eff}}$ , as is illustrated by the following example.

**EXAMPLE 4.** Continue with Example 3. For the costs of preferred suppliers, we assume that they are uniformly distributed on  $[2, 3]$  instead.<sup>3</sup> Then we can derive the optimal scoring rule  $\Delta^{\text{opt}} = 3/2 > \Delta^{\text{eff}} = 3/4$ .

This example illustrates that it is not always profitable for the buyer to discriminate against the preferred suppliers. The reason is that, in our setting, suppliers' contributions in the bidding competition are determined by both their success probabilities and their cost distributions. When preferred suppliers, who have an advantage in success probability, are disadvantaged in terms of cost distribution, they may behave aggressively in the bidding competition. As a result, a preferred supplier does not necessarily contribute less to the buyer than its nonpreferred counterpart who has the same expected total cost.

## 7. Extensions and Discussion

In this section, we consider relaxing some of the model assumptions.

### 7.1. Degree of Failure

In the baseline model, we focus on the setting with a zero-or-one outcome: The winner eventually either succeeds or fails to meet the requirements specified by the buyer. In other words, the outcome of the project is either a success or a failure. The punishment scheme is based on such an outcome. In some

other cases, the buyer may also care about the degree (if measurable) to which the winner does not meet the requirements. One example of measurable degree is the delay of delivery of the project. The buyer is concerned not only about whether the winner can accomplish the project on time, but also about how soon the winner can complete the project if it passes the due date. In public sector procurement, such as infrastructure construction projects, the degree of delay affects the perceived loss of social welfare. In private sector procurement, the degree of delay could affect business opportunity, such as the loss of consumer goodwill (e.g., resulting from delay in receipt of the goods requested), and the timing of entering a new market.

We denote the requirement level (e.g., quality level) specified by the buyer as  $r$ ,  $r \in (0, 1)$ . Instead of assuming that suppliers can meet the requirement or not, we let the outcome level  $x$  of supplier  $i$  satisfy some distribution characterized by a probability density function  $g_i(x)$ ,  $x \in [0, 1]$ . If the realized outcome  $x \leq r$ , the project meets the requirement; otherwise, it does not, and the difference  $x - r$  measures the degree of failure. We follow the same auction model as in the baseline case except that the penalty  $t$  is now a unit payment for unit degree of failure. Similarly, we can formulate the payoff function for suppliers as

$$U(b, t; c, g) = \left( b - c - t \int_r^1 (x - r)g(x) dx \right) \Pr(\text{win}).$$

For the buyer, the potential loss is generally a function of the degree of the failure. Therefore, instead of a constant  $z$ , we introduce a loss function  $\zeta(x - r)$ , which is increasing in its argument. Hence, the expected payoff for the buyer when bidder  $i$  wins the contract is

$$V(b_i, t_i, g_i) = v - b_i - \int_r^1 \zeta(x - r)g_i(x) dx \\ + t_i \int_r^1 (x - r)g_i(x) dx.$$

Following the same approach, all the analysis in the baseline case can be conducted and similar results can be derived. In particular, if the loss function is linear (i.e.,  $\zeta(x - r) = z(x - r)$ ), we can simply denote  $q_i \equiv \int_r^1 (x - r)g_i(x) dx$ , and all the results are the same as in the baseline case. The only difference is the interpretation of  $q$ : In the baseline case,  $q$  is the probability of failing to meet the requirements, and here  $q$  is the expected degree of failure.

<sup>3</sup>Notice that in this case, it is the preferred suppliers with cost within  $[2, 1 + \Delta]$  who are comparable with the nonpreferred suppliers with cost within  $[2 - \Delta, 1]$ , provided that  $\Delta > 1$ .

## 7.2. Multiple Types of Suppliers

The basic insight derived from our baseline setting holds not just for two supplier types but also for multiple types of suppliers. Now we suppose there are  $k$  types of suppliers, indexed by  $\theta = 1, 2, \dots, k$ , and the corresponding failure probabilities are  $q_1, q_2, \dots, q_k$ . Without loss of generality, we assume  $q_1 < q_2 < \dots < q_k$ . We denote the probability of any individual supplier's being type  $\theta$  as  $\alpha_\theta$  and the associated cost distribution as  $F_\theta(c) \equiv \int_c^{\bar{c}} f(x, q_\theta) dx$ . Everything else follows the baseline setting.

Lemma 1 continues to hold because the analysis is not affected by the number of supplier types. So suppliers place bids on penalty,  $t^*(q_\theta)$ , along the same principle based on their private information of their failure probabilities. Similar to  $\Delta$ , for  $\theta = 2, \dots, k$ , we now introduce  $\Delta_\theta$ :

$$\Delta_\theta \equiv [\Lambda(t^*(q_{\theta-1})) - q_{\theta-1}t^*(q_{\theta-1})] - [\Lambda(t^*(q_\theta)) - q_\theta t^*(q_\theta)].$$

We also define  $m_\theta(c)$  and  $c_\theta^*$  such that  $S(b_\theta(c), t^*(q_\theta)) = S(b_{\theta-1}(m_\theta(c)), t^*(q_{\theta-1}))$ . Based on these notations, we can formulate the equilibrium probability of winning for a supplier with type  $\theta$  and cost  $c$ :

$$\rho_\theta(c) = \left[ \sum_{i=1}^{\theta-1} \alpha_i [1 - F_i(m_{i+1}(m_{i+2}(\dots m_\theta(c))))] + \alpha_\theta [1 - F_\theta(c)] \right. \\ \left. + \sum_{i=\theta+1}^k \alpha_i [1 - F_i(m_i^{-1}(m_{i-1}^{-1}(\dots m_{\theta+1}^{-1}(c))))] \right]^{n-1}.$$

We can then obtain  $k$  equilibrium bidding functions on cost in the same way as in Proposition 1, one for each type. Specifically,

$$b_k(c) = c + q_k t^*(q_k) + \frac{\int_c^{\bar{c}} \rho_k(x) dx}{\rho_k(c)}, \\ b_\theta(c) = c + q_\theta t^*(q_\theta) + \frac{\sum_{i=\theta+1}^k \int_{c_i^*}^{\bar{c}} \rho_i(x) dx + \int_c^{\bar{c}} \rho_\theta(x) dx}{\rho_\theta(c)}, \\ (\theta = 1, \dots, k-1),$$

where  $m_\theta(c) = c + \Delta_\theta$  and  $c_\theta^* = \bar{c} - \Delta_\theta$ .

Analogous to the two-type case, any scoring rules satisfying  $\Delta_\theta = z(q_\theta - q_{\theta-1})$ ,  $\theta = 2, 3, \dots, k$ , are socially efficient. The optimal scoring rule can also be characterized in a similar way (with more complexity), and basic intuition continues to hold. For example, under similar regularity and distributional upgrade

conditions, it is revenue-maximizing for the buyer to under-reward the penalty payment from the suppliers with the lowest failure rate (compared to the suppliers with the highest failure rate).

## 8. Conclusion

In this paper, we focus on an unexplored procurement setting in which suppliers' ability to successfully accomplish the project is important to the buyer. We consider a model in which suppliers differ in both their costs and in their types in terms of success probabilities. The buyer introduces a contingent contract on the outcome of the project to further screen suppliers beyond the basis of cost. In particular, suppliers are asked to specify a penalty if they fail to deliver the project as required. We find that an easy-to-implement quasi-linear scoring rule can effectively separate suppliers in regard to their types. In equilibrium, suppliers bid different amounts of monetary penalty based on their own types. Suppliers' bids on the cost take into account the possible penalty in case of failure, such that the inferred information regarding suppliers' success probability does not directly benefit the buyer. However, the inferred information can be used to find the most suitable supplier or even to leverage the competition. A properly designed scoring rule can easily implement an efficient allocation in which the supplier with the lowest expected total cost to the buyer gets awarded the contract, and the inferred information is essentially internalized. To minimize the total procurement cost, the buyer may or may not under-reward suppliers with high success probability, depending on the balance between suppliers' success probabilities and the associated cost distributions. Because suppliers differ in two dimensions, the conventional principle of under-rewarding the high type can work only if the suppliers with high success probability also have distributional advantage on their cost; otherwise, over-rewarding the high type could be optimal to minimize the procurement cost. In addition, to promote the competition between different types of suppliers, it is optimal for the buyer to always let at least some suppliers with low success probability be able to win over some of those with high success probability, even though such allocation may appear rather inefficient when the difference in their success probabilities is huge.

Our analysis has several implications. First, we illustrate the importance and effectiveness of introducing contingent contracts in screening suppliers and call for procurement managers' attention to properly use such an economic device in auction design. Most noncommodity procurements are associated with risk to some extent; suppliers may not deliver the project on time, or the delivered project might not satisfy the requirement prespecified. Buyers are thus encouraged to recognize such risks and to take some precaution in choosing the contractor *ex ante* to avoid the considerable cost of possible lawsuits or renegotiation. We show that a simple scoring rule based on suppliers' bids on cost and penalty in case of failure can effectively help screen suppliers without any operational cost associated with any physical screening (e.g., visiting the supplier and checking its facilities). Compared to standard auctions, the procurement auctions with contingent contracts can significantly improve both social welfare and the buyer's payoff.

Second, different designs of the scoring rule are prescribed. The scoring rule to maximize the total social welfare is remarkably simple and easy to implement, despite the complexity of suppliers' strategic behavior. Any scoring rule that essentially chooses the supplier with the lowest total expected cost to the buyer can achieve the goal. Such a rule is also very easy to implement: The buyer only needs to estimate the distribution of suppliers' success probabilities and its own loss from a failure. To minimize the procurement cost, the buyer should adjust the scoring rule. Such a rule may favor or disfavor suppliers with high success probability. When suppliers with high and low suitability have the same cost distribution, the buyer may obtain a lower procurement cost by favoring the suppliers with low suitability. When the suppliers with lower success probability have some advantage in their cost distribution, the best design may favor those suppliers less and possibly even disfavor them. Such results suggest that we cannot automatically assume that the low-type suppliers should be favored in a cost-minimizing design. It is also worth noting that the design of scoring rules we prescribe are independent of the forms of scoring rules. Therefore, many commonly used classes of scoring rules (e.g., square-root and logarithmic

scoring rules) with proper parameters can serve the purpose of maximizing social welfare or minimizing procurement cost (within the class of quasi-linear scoring rules), and the choice of the specific scoring rule should depend on other criteria.

Third, our results also have implications for suppliers' bidding strategies. For example, suppliers with high success probability should signal their confidence by bidding aggressively on the penalty in case of a failure. Meanwhile, they should anticipate the possible penalty and take that into account when formulating the bid on cost.

Several other issues may be interesting for future research. First, the moral hazard problem of suppliers can be discussed and incorporated in a future study. In the current work, we focus on the adverse selection problem in which suppliers differ in their types, which are their inherent nature, and hence have no control over the cost or probability of success. Although the framework of adverse selection can address a class of problems and serves as a good starting point, further investigation on suppliers' incentive to exert effort to influence the cost and/or success probability can be a good complement to the current study. Second, it would be interesting to further examine the format of contingent contracts and its effect. For example, we may allow suppliers to propose different contingent contract formats to reflect their types.

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### Appendix

#### A.1. Proof of Proposition 1

PROOF. By the envelope theorem, we have  $(dU(b_h(c), t^*(q_h); c, q_h))/dc = \partial U/\partial c = -\rho_h(c)$  from Equation (6) by noting  $\Pr(\text{win}) = \rho_h(c)$  in equilibrium. Notice the boundary condition  $U(b_h(\bar{c}), t^*(q_h); \bar{c}, q_h) = 0$  (i.e., the nonpreferred supplier with the highest cost earns zero profit in equilibrium). So we have

$$U(b_h(c), t^*(q_h); c, q_h) = \int_c^{\bar{c}} \rho_h(x) dx.$$

Combining Equation (6) in equilibrium (so  $\Pr(\text{win}) = \rho_h(c)$ ) with the above payoff function, we can obtain the bidding function  $b_h(c)$ .

Applying the envelope theorem again to the preferred suppliers' equilibrium payoff, we can derive

$$U(b_l(c), t^*(q_l); c, q_l) = U(b_l(\bar{c}), t^*(q_l); \bar{c}, q_l) + \int_c^{\bar{c}} \rho_l(x) dx$$

and

$$b_l(c) = c + q_l t^*(q_l) + \frac{U(b_l(\bar{c}), t^*(q_l); \bar{c}, q_l) + \int_c^{\bar{c}} \rho_l(x) dx}{\rho_l(c)}.$$

Then notice that a nonpreferred supplier with cost  $c$  ties with a preferred supplier with cost  $m(c)$ . We have

$$\begin{aligned} c + q_h t^*(q_h) + \frac{\int_c^{\bar{c}} \rho_h(x) dx}{\rho_h(c)} - \Lambda(t^*(q_h)) \\ &= m(c) + q_l t^*(q_l) \\ &\quad + \frac{U(b_l(\bar{c}), t^*(q_l); \bar{c}, q_l) + \int_{m(c)}^{\bar{c}} \rho_l(x) dx}{\rho_l(m(c))} - \Lambda(t^*(q_l)) \\ &= m(c) + q_l t^*(q_l) \\ &\quad + \frac{U(b_l(\bar{c}), t^*(q_l); \bar{c}, q_l) + \int_c^{c^*} \rho_h(x) m'(x) dx}{\rho_h(c)} \\ &\quad - \Lambda(t^*(q_l)). \end{aligned} \quad (17)$$

Multiplying both sides by  $\rho_h(c)$  and taking the first-order derivative with respect to  $c$ , we have

$$\begin{aligned} c \rho_h'(c) + \rho_h(c) + q_h t^*(q_h) \rho_h'(c) - \rho_h(c) - \Lambda(t^*(q_h)) \rho_h'(c) \\ &= m'(c) \rho_h(c) + m(c) \rho_h'(c) + q_l t^*(q_l) \rho_h'(c) \\ &\quad - \rho_h(c) m'(c) - \Lambda(t^*(q_l)) \rho_h'(c). \end{aligned}$$

Therefore,  $m(c) = c + \Delta$  and thus  $c^* = \bar{c} - \Delta$ .

Substituting  $m(c)$  into Equation (17), we have

$$\begin{aligned} c + q_h t^*(q_h) + \frac{\int_c^{\bar{c}} \rho_h(x) dx}{\rho_h(c)} - \Lambda(t^*(q_h)) \\ &= c + \Delta + q_l t^*(q_l) + \frac{U(b_l(\bar{c}), t^*(q_l); \bar{c}, q_l) + \int_c^{c^*} \rho_h(x) dx}{\rho_h(c)} \\ &\quad - \Lambda(t^*(q_l)). \end{aligned}$$

Therefore, we have

$$U(b_l(\bar{c}), t^*(q_l); \bar{c}, q_l) = \int_{c^*}^{\bar{c}} \rho_h(x) dx.$$

To show that the above bidding functions  $b_h(c)$  and  $b_l(c)$  are equilibrium bidding strategies, we need to verify that suppliers have no profitable deviation. For a nonpreferred supplier with cost  $c$ , given everyone else's bidding strategy,

deviating from the presumed bidding strategy is equivalent to reporting a different cost  $c'$ . The payoff difference is

$$\begin{aligned} U(b_h(c'), t^*(q_h); c, q_h) - U(b_h(c), t^*(q_h); c, q_h) \\ &= (c' - c) \rho_h(c') - \int_c^{c'} \rho_h(x) dx. \end{aligned}$$

Because  $\rho_h(c)$  is decreasing in  $c$ , the payoff difference is negative for any  $c' \neq c$ . Therefore, deviation is nonprofitable for the nonpreferred bidders. Similar arguments apply to the preferred bidders.  $\square$

## A.2. Proof of Proposition 2

PROOF. As discussed in the text,  $\Delta = z(q_h - q_l)$  can ensure ex post efficiency. We next show that  $\Delta = z(q_h - q_l)$  can also guarantee ex ante efficiency. First, note that  $\rho_l(c) = \rho_h(c - \Delta)$  and

$$\left. \frac{d\rho_h(c)}{d\Delta} \right|_{c-\Delta} = -\frac{\alpha}{1-\alpha} \frac{f_l(c)}{f_h(c-\Delta)} \frac{d\rho_l(c)}{d\Delta}. \quad (18)$$

Notice that in the expression of the expected social welfare Equation (10), the scoring rule is fully represented by  $\Delta$ . Take the first-order derivative of Equation (10) with respect to  $\Delta$ ,

$$\begin{aligned} n\alpha \int_{\underline{c}}^{\bar{c}} (v - q_l z - c) \frac{d\rho_l(c)}{d\Delta} f_l(c) dc \\ + n(1-\alpha) \int_{\underline{c}}^{\bar{c}} (v - q_h z - c) \frac{d\rho_h(c)}{d\Delta} f_h(c) dc. \end{aligned}$$

Noting that  $d\rho_h(c)/d\Delta = 0$  for  $c > c^* = \bar{c} - \Delta$  and that  $d\rho_l(c)/d\Delta = 0$  for  $c < m^{-1}(\underline{c}) = \underline{c} + \Delta$ , we can reorganize the above as

$$\begin{aligned} \alpha \int_{\underline{c}+\Delta}^{\bar{c}} (v - q_l z - c) \frac{d\rho_l(c)}{d\Delta} f_l(c) dc \\ + (1-\alpha) \int_{\underline{c}}^{\bar{c}-\Delta} (v - q_h z - c) \frac{d\rho_h(c)}{d\Delta} f_h(c) dc \\ &= \alpha \int_{\underline{c}+\Delta}^{\bar{c}} (v - q_l z - c) \frac{d\rho_l(c)}{d\Delta} f_l(c) dc \\ &\quad - \alpha \int_{\underline{c}+\Delta}^{\bar{c}} (v - q_h z - c + \Delta) \frac{d\rho_l(c)}{d\Delta} f_l(c) dc \\ &= \alpha \int_{\underline{c}+\Delta}^{\bar{c}} (q_h z - q_l z - \Delta) \frac{d\rho_l(c)}{d\Delta} f_l(c) dc, \end{aligned}$$

where the first equality is because of integration by substitution and Equation (18). Because  $d\rho_l(c)/d\Delta > 0$ , the above first-order derivative is positive if  $\Delta < z(q_h - q_l)$  and negative if  $\Delta > z(q_h - q_l)$ . So  $\Delta = z(q_h - q_l)$  maximizes the social welfare.  $\square$

## A.3. The Derivation of Expected Payoff

The buyer's expected payoff from a supplier is equal to the value created upon winning minus the supplier's expected payoff:

$$(v - q_j z - c) \rho_j(c) - U(b_j(c), t^*(q_j); c, q_j),$$

where  $j \in \{h, l\}$ . Notice  $U(b_h(c), t^*(q_h); c, q_h) = \int_c^{\bar{c}} \rho_h(x) dx$  and  $U(b_l(c), t^*(q_l); c, q_l) = \int_{c^*}^{\bar{c}} \rho_h(x) dx + \int_c^{\bar{c}} \rho_l(x) dx$  from the proof of Proposition 1. Then, the payoff from one supplier (with probability  $\alpha$  being preferred and with probability  $(1 - \alpha)$  being nonpreferred) is

$$\begin{aligned} & \alpha E[(v - q_l z - c)\rho_l(c) - U(b_l(c), t^*(q_l); c, q_l)] \\ & + (1 - \alpha) E[(v - q_h z - c)\rho_h(c) - U(b_h(c), t^*(q_h); c, q_h)] \\ & = \alpha \int_{\underline{c}}^{\bar{c}} \left[ (v - q_l z - c)\rho_l(c) - \int_{c^*}^{\bar{c}} \rho_h(x) dx - \int_c^{\bar{c}} \rho_l(x) dx \right] f_l(c) dc \\ & + (1 - \alpha) \int_{\underline{c}}^{\bar{c}} \left[ (v - q_h z - c)\rho_h(c) - \int_c^{\bar{c}} \rho_h(x) dx \right] f_h(c) dc \\ & = \alpha \int_{\underline{c}}^{\bar{c}} \left[ v - q_l z - c - \frac{F_l(c)}{f_l(c)} \right] \rho_l(c) f_l(c) dc - \alpha \int_{c^*}^{\bar{c}} \rho_h(x) dx \\ & + (1 - \alpha) \int_{\underline{c}}^{\bar{c}} \left[ v - q_h z - c - \frac{F_h(c)}{f_h(c)} \right] \rho_h(c) f_h(c) dc \\ & = \frac{1}{n} v - \alpha \int_{\underline{c}}^{\bar{c}} \left[ q_l z + c + \frac{F_l(c)}{f_l(c)} \right] \rho_l(c) f_l(c) dc - \alpha \int_{c^*}^{\bar{c}} \rho_h(x) dx \\ & - (1 - \alpha) \int_{\underline{c}}^{\bar{c}} \left[ q_h z + c + \frac{F_h(c)}{f_h(c)} \right] \rho_h(c) f_h(c) dc, \end{aligned}$$

where the second equality is achieved by exchanging the integration order and the third equality is because  $\alpha \int_{\underline{c}}^{\bar{c}} \rho_l(c) f_l(c) dc + (1 - \alpha) \int_{\underline{c}}^{\bar{c}} \rho_h(c) f_h(c) dc = 1/n$  (i.e., the ex ante expected probability of winning for any bidder is  $1/n$ ). The total expected payoff for the buyer is thus  $n$  times the above.

#### A.4. Proof of Proposition 3

PROOF. Taking the first-order derivative of the expected payoff Equation (13) with respect to  $\Delta$  yields the following:

$$\begin{aligned} & -\alpha \rho_h(\bar{c} - \Delta) - n\alpha \int_{\underline{c}}^{\bar{c}} \left[ q_l z + c + \frac{F_l(c)}{f_l(c)} \right] \frac{d\rho_l(c)}{d\Delta} f_l(c) dc \\ & - n(1 - \alpha) \int_{\underline{c}}^{\bar{c}} \left[ q_h z + c + \frac{F_h(c)}{f_h(c)} \right] \frac{d\rho_h(c)}{d\Delta} f_h(c) dc. \end{aligned}$$

Noting that  $d\rho_h(c)/d\Delta = 0$  for  $c > \bar{c} - \Delta$  and  $d\rho_l(c)/d\Delta = 0$  for  $c < \underline{c} + \Delta$ , we can reorganize the above as

$$\begin{aligned} & -\alpha \rho_h(\bar{c} - \Delta) - \alpha \int_{\underline{c} + \Delta}^{\bar{c}} \left[ q_l z + c + \frac{F_l(c)}{f_l(c)} \right] \frac{d\rho_l(c)}{d\Delta} f_l(c) dc \\ & - (1 - \alpha) \int_{\underline{c}}^{\bar{c} - \Delta} \left[ q_h z + c + \frac{F_h(c)}{f_h(c)} \right] \frac{d\rho_h(c)}{d\Delta} f_h(c) dc \\ & = -\alpha \rho_h(\bar{c} - \Delta) - \alpha \int_{\underline{c} + \Delta}^{\bar{c}} \left[ q_l z + c + \frac{F_l(c)}{f_l(c)} \right] \frac{d\rho_l(c)}{d\Delta} f_l(c) dc \\ & + \alpha \int_{\underline{c} + \Delta}^{\bar{c}} \left[ q_h z + c - \Delta + \frac{F_h(c - \Delta)}{f_h(c - \Delta)} \right] \frac{d\rho_l(c)}{d\Delta} f_l(c) dc \\ & = \alpha \int_{\underline{c} + \Delta}^{\bar{c}} \left[ q_h z - q_l z - \Delta + \frac{F_h(c - \Delta)}{f_h(c - \Delta)} - \frac{F_l(c)}{f_l(c)} \right] \frac{d\rho_l(c)}{d\Delta} f_l(c) dc \\ & - \alpha \rho_h(\bar{c} - \Delta), \end{aligned}$$

where the first equality is because of integration by substitution and Equation (18). Notice that the first-order derivative at  $\Delta = \bar{c} - \underline{c}$  is negative, and thus  $\Delta = \bar{c} - \underline{c}$  cannot be a corner solution. Therefore, the optimal scoring rule is either at the other corner  $\Delta = 0$  (as we can show,  $\Delta$  cannot be negative) or characterized by the above first-order derivative being zero.

(Notice that any  $\Delta > \bar{c} - \underline{c}$  is suboptimal. This is because when  $\Delta \geq \bar{c} - \underline{c}$ , based on Equation (9), the buyer's expected payoff can be similarly formulated as

$$\begin{aligned} & v - n\alpha \left[ \int_{\underline{c}}^{\bar{c}} \left[ q_l z + c + \frac{F_l(c)}{f_l(c)} \right] \rho_l(c) f_l(c) dc \right. \\ & \left. + \int_{\underline{c}}^{\bar{c}} \rho_h(c) dc + (\Delta - \bar{c} + \underline{c}) \rho_l(\bar{c}) \right] \\ & - n(1 - \alpha) \int_{\underline{c}}^{\bar{c}} \left[ q_h z + c + \frac{F_h(c)}{f_h(c)} \right] \rho_h(c) f_h(c) dc, \end{aligned}$$

which, by noting that  $\rho_l(c)$  and  $\rho_h(c)$  are independent of  $\Delta$ , is decreasing in  $\Delta$ .)

If  $F_h(c)$  satisfies the regularity condition and  $F_l(c)$  is a distributional upgrade of  $F_h(c)$ ,

$$\frac{F_h(c - \Delta)}{f_h(c - \Delta)} < \frac{F_h(c)}{f_h(c)} \leq \frac{F_l(c)}{f_l(c)}.$$

Because  $d\rho_l(c)/d\Delta > 0$  and  $\rho_h(\bar{c} - \Delta) > 0$ , the above first-order derivative is negative for all  $\Delta \geq z(q_h - q_l)$ . So  $\Delta^{\text{opt}} < z(q_h - q_l)$ .  $\square$

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