Online Appendix for ISR Manuscript

"Strategic Intellectual Property Sharing: Competition on an Open Technology Platform Under Network Effects"

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A Proofs of Main and Supporting Results

A.1 Proof of Monopolistic Pricing (§4.1)

Proof. When the incumbent is a monopoly in the market, the boundary consumer who is indifferent between purchasing from the incumbent and not purchasing at all, $\tilde{\theta}$, can be formulated as $\left(\tilde{\theta}+\gamma N\right)-p_1=0$. Therefore, $\tilde{\theta}=p_1-\gamma N$. In a rational expectations equilibrium (REE), consumers rationally anticipate the network size, so N and $\tilde{\theta}$ need to be solved simultaneously. Suppose $0<\tilde{\theta}<1$, then $N=N_1=1-\tilde{\theta}$. As a result, $\tilde{\theta}$ can be solved as $\tilde{\theta}=\frac{p_1-\gamma}{1-\gamma}$. As we can easily check, $\frac{p_1-\gamma}{1-\gamma}>0$ if and only if $p_1>\gamma$; $\frac{p_1-\gamma}{1-\gamma}<1$ if and only if $p_1<1$. Therefore, we can summarize the demand function for the monopolistic incumbent as follows.

$$N_1\left(p_1\right)=\begin{cases}1, & p_1\leq\gamma;\\ 1-\frac{p_1-\gamma}{1-\gamma}, & \gamma<p_1<1;\\ 0, & 1\leq p_1.\end{cases} \tag{A.1}$$

The incumbent sets p_1 to maximize its profit function $\pi_1\left(p_1\right)=p_1N_1\left(p_1\right)$. It is easy to show that the optimal p_1 must fall in $[\gamma,1)$. The first order condition yields $\hat{p}_1=\frac{1}{2}$. Therefore, if $\gamma<\frac{1}{2}$, $\hat{p}_1>\gamma$, so $p_1^*=\hat{p}_1=\frac{1}{2}$; if $\gamma\geq\frac{1}{2}$, $\hat{p}_1\leq\gamma$, so p_1^* takes the corner solution, i.e., $p_1^*=\gamma$. □

A.2 Proof of Proposition 1

Proof. To derive the pricing equilibrium, we need to examine three stages of strategic decisions (i.e., the last three stages of the whole model). Along the line of backward induction, we derive the pricing equilibrium in the following three steps: (1) first, determine the demands for both firms' products as functions of the prices, $N_i\left(p_1, p_2\right)$ $(i = 1, 2)$; (2) next, determine the incumbent's best response in pricing as a function of the entrant's price, $p_1^*\left(p_2\right)$; (3) finally, determine the optimal price of the entrant, p_2^*. Throughout the analysis for this proposition, we take the entrant's product quality q and the strength of network effects γ as given, and discuss the equilibrium outcomes when these parameters take different values.

(1) We first derive the demand functions $N_i\left(p_1, p_2\right)$ $(i = 1, 2)$ given p_1 and p_2.

Consumers choose among three options: purchasing from the incumbent, purchasing from the entrant, and not purchasing at all. The boundary consumer who is indifferent between purchasing from the incumbent and purchasing from the entrant, $\tilde{\theta}_{12}$, can be derived by solving $\tilde{\theta}_{12}+\gamma N-p_1 = \left(\tilde{\theta}_{12}+\gamma N\right) q - p_2$, which yields $\tilde{\theta}_{12} = \frac{p_1-p_2}{1-q} - \gamma N$, where $N = N_1 + N_2$. Similarly, the boundary consumer indifferent between purchasing from the entrant and not purchasing at all is $\tilde{\theta}_2 = \frac{p_2}{q} - \gamma N$; the boundary consumer indifferent between purchasing from the incumbent and not purchasing at all is $\tilde{\theta}_1 = p_1 - \gamma N$. As we can easily show, all consumers with $\theta > \tilde{\theta}_{12}$ prefer purchasing from the incumbent to purchasing from the entrant, and vice versa; likewise, all consumers with $\theta < \tilde{\theta}_2$ (or $\tilde{\theta}_1$) prefer purchasing nothing to purchasing from the entrant (or incumbent), and vice versa.

In REE, consumers form rational expectations about the total network size N when making purchase decisions. Therefore, $\left\{\tilde{\theta}_{12}, \tilde{\theta}_2, \tilde{\theta}_1\right\}$ need to be simultaneously solved with N. Note that $\{N_1, N_2\}$ and hence N all depend on the relative magnitude of $\tilde{\theta}_{12}, \tilde{\theta}_2, \tilde{\theta}_1$, and the bounds of θ's range $[0, 1]$. Comparing these relative magnitudes and solving $\left\{\tilde{\theta}_{12}, \tilde{\theta}_2, \tilde{\theta}_1, N\right\}$ simultaneously lead to different demand cases under different parameter conditions. For example, suppose $0 < \tilde{\theta}_2 \left(< \tilde{\theta}_1\right) < \tilde{\theta}_{12} < 1$, then $N_1 = 1 - \tilde{\theta}_{12}$, $N_2 = \tilde{\theta}_{12} - \tilde{\theta}_2$, and as a result, $N = N_1 + N_2 = 1 - \tilde{\theta}_2$. Substituting $N = 1 - \tilde{\theta}_2$ into $\tilde{\theta}_2 = \frac{p_2}{q} - \gamma N$, we can solve $\tilde{\theta}_2$ as $\tilde{\theta}_2 = \frac{p_2 - q\gamma}{q(1-\gamma)}$. Consequently, $N = \frac{q-p_2}{q(1-\gamma)}$, and $\tilde{\theta}_{12} = \frac{p_1-p_2}{1-q} - \gamma\frac{q-p_2}{q(1-\gamma)}$, $\tilde{\theta}_1 = p_1 - \gamma\frac{q-p_2}{q(1-\gamma)}$. We then need to verify under what conditions $0 < \tilde{\theta}_2 \left(< \tilde{\theta}_1\right) < \tilde{\theta}_{12} < 1$ holds. Solving the inequalities after substituting the solutions of $\left\{\tilde{\theta}_{12}, \tilde{\theta}_2, \tilde{\theta}_1\right\}$, we arrive at the conditions: $q\gamma < p_2 < q$, and $\frac{p_2}{q} < p_1 < \frac{(q-\gamma)p_2 + q(1-q)}{q(1-\gamma)}$.

In a similar way to the case of $0 < \tilde{\theta}_2 \left(< \tilde{\theta}_1\right) < \tilde{\theta}_{12} < 1$ analyzed above, we can exhaust all cases of different relative magnitudes of $\tilde{\theta}_{12}, \tilde{\theta}_2, \tilde{\theta}_1$ and the bounds $[0,1]$, which gives us the demands in different regions, as summarized in Table A1.

Table A1: Demands Given Both Firms' Prices p_1 and p_2

Cases	Conditions	$N_1(p_1,p_2)$	$N_2(p_1,p_2)$	$N(p_1,p_2)$
(A)	$0 < p_2 \leq q\gamma$:			
(A1)	$0 < p_1 \leq p_2 + (1-q)\gamma$;	1	0	1
(A2)	$p_2 + (1-q)\gamma < p_1 < p_2 + (1-q)(1+\gamma)$;	$1+\gamma-\frac{p_1-p_2}{1-q}$	$\frac{p_1-p_2}{1-q}-\gamma$	1
(A3)	$p_2 + (1-q)(1+\gamma) \leq p_1 < 1+\gamma$	0	1	1
(B)	$q\gamma < p_2 < q$:			
(B1)	$0 < p_1 \leq \gamma$;	1	0	1
(B2)	$\gamma < p_1 \leq \frac{p_2}{q}$;	$\frac{1-p_1}{1-\gamma}$	0	$\frac{1-p_1}{1-\gamma}\ (<1)$
(B3)	$\frac{p_2}{q} < p_1 < \frac{(q-\gamma)p_2+q(1-q)}{q(1-\gamma)}$;	$1-\frac{p_1-p_2}{1-q}+\gamma\frac{q-p_2}{q(1-\gamma)}$	$\frac{p_1-p_2}{1-q}-\frac{p_2}{q}$	$\frac{q-p_2}{q(1-\gamma)}\ (<1)$
(B4)	$\frac{(q-\gamma)p_2+q(1-q)}{q(1-\gamma)} \leq p_1 < 1+\gamma$	0	$\frac{q-p_2}{q(1-\gamma)}$	$\frac{q-p_2}{q(1-\gamma)}\ (<1)$
(C)	$q \leq p_2 < q(1+\gamma)$:			
(C1)	$0 < p_1 \leq \gamma$;	1	0	1
(C2)	$\gamma < p_1 \leq 1$;	$\frac{1-p_1}{1-\gamma}$	0	$\frac{1-p_1}{1-\gamma}\ (<1)$
(C3)	$1 < p_1 < 1+\gamma$	0	0	0

(2) We next derive the incumbent's best response function $p_1^*(p_2)$.

Based on the demands derived in Table A1, we can formulate the profit function of the incumbent given the entrant's price, $\pi_1(p_1;p_2)$. For example, according to Case (A) in Table A1, when $0 < p_2 \leq q\gamma$, we have

$$\pi_1(p_1;p_2) = \begin{cases} p_1, & 0 < p_1 \leq p_2 + (1-q)\gamma; \\ p_1\left(1+\gamma-\frac{p_1-p_2}{1-q}\right), & p_2+(1-q)\gamma < p_1 < p_2+(1-q)(1+\gamma); \\ 0, & p_2+(1-q)(1+\gamma) \leq p_1 < 1+\gamma. \end{cases} \tag{A.2}$$

It is easy to see that any $p_1 < p_2 + (1-q)\gamma$ or $p_1 > p_2 + (1-q)(1+\gamma)$ cannot be the optimal price for the incumbent. Therefore, we only need to focus on the second segment in (A.2). The first order condition yields the solution $\hat{p}_1 = \frac{1}{2}[(1-q)(1+\gamma)+p_2]$. We then need to compare $\hat{p}_1$ against the two bounds of that segment, $p_2 + (1-q)\gamma$ and $p_2 + (1-q)(1+\gamma)$. Note that $\hat{p}_1 < p_2 + (1-q)(1+\gamma)$ automatically holds, and $\hat{p}_1 > p_2 + (1-q)\gamma$ if and only if $p_2 <$

$(1-q)(1-\gamma)$. Because $0 < p_2 \leq q\gamma$ under Case (A), we also need to compare $(1-q)(1-\gamma)$ with $q\gamma$: $(1-q)(1-\gamma) < q\gamma$ if and only if $q > 1-\gamma$. Altogether, we have the following three subcases: (a) if $q > 1-\gamma$ and $0 < p_2 < (1-q)(1-\gamma)\,(< q\gamma)$, then $p_1^*(p_2) = \frac{1}{2}[(1-q)(1+\gamma)+p_2]$, and the demands fall into Case (A2) as in Table A1; (b) if $q > 1-\gamma$ and $(1-q)(1-\gamma) < p_2 \leq q\gamma$, then $p_1^*(p_2) = p_2 + (1-q)\gamma$, and the demands fall into Case (A1) as in Table A1 (in fact, the intersecting bound between Cases (A1) and (A2)); (c) if $q \leq 1-\gamma$ and $0 < p_2 \leq q\gamma\,(\leq (1-q)(1-\gamma))$, then $p_1^*(p_2) = \frac{1}{2}[(1-q)(1+\gamma)+p_2]$, and the demands fall into Case (A2) as in Table A1.

Table A2: The Incumbent's Best Response Function $p_1^*(p_2)$

Cases	Conditions	$p_1^*(p_2)$	Demand Cases (as in Table A1)
(1)	$0 < \gamma < \frac{1}{2}$, $0 < q \leq \gamma\,(< 1-\gamma)$:		
(1a)	$0 < p_2 \leq q\gamma$;	$\frac{(1-q)(1+\gamma)+p_2}{2}$	(A2)
(1b)	$q\gamma < p_2 < q\frac{(1-q)-\sqrt{(1-q)(1-\gamma)}}{\gamma-q}$;	$\frac{(q-\gamma)p_2+q(1-q)}{2q(1-\gamma)}$	(B3)
(1c)	$q\frac{(1-q)-\sqrt{(1-q)(1-\gamma)}}{\gamma-q} \leq p_2 < q$	$\frac{1}{2}$	(B2)
(2)	$0 < \gamma < \frac{1}{2}$, $\gamma < q \leq 1-\gamma$:		
(2a)	$0 < p_2 \leq q\gamma$;	$\frac{(1-q)(1+\gamma)+p_2}{2}$	(A2)
(2b)	$q\gamma < p_2 \leq q\frac{(1-q)}{2-q-\gamma}$;	$\frac{(q-\gamma)p_2+q(1-q)}{2q(1-\gamma)}$	(B3)
(2c)	$q\frac{(1-q)}{2-q-\gamma} < p_2 < q$	$\min\left\{\frac{p_2}{q}, \frac{1}{2}\right\}$	(B2) / (B3)
(3)	$0 < \gamma < \frac{1}{2}$, $(\gamma <)\,1-\gamma < q < 1$:		
(3a)	$0 < p_2 \leq (1-q)(1-\gamma)$;	$\frac{(1-q)(1+\gamma)+p_2}{2}$	(A2)
(3b)	$(1-q)(1-\gamma) < p_2 \leq q\gamma$;	$p_2 + (1-q)\gamma$	(A1) / (A2)
(3c)	$q\gamma < p_2 < q$	$\min\left\{\frac{p_2}{q}, \frac{1}{2}\right\}$	(B2) / (B3)
(4)	$\frac{1}{2} \leq \gamma < 1$, $0 < q < 1-\gamma\,(\leq \gamma)$:		
(4a)	$0 < p_2 \leq q\gamma$;	$\frac{(1-q)(1+\gamma)+p_2}{2}$	(A2)
(4b)	$q\gamma < p_2 < q\frac{(1-q)-2(1-\gamma)\sqrt{(1-q)\gamma}}{(\gamma-q)}$;	$\frac{(q-\gamma)p_2+q(1-q)}{2q(1-\gamma)}$	(B3)
(4c)	$q\frac{(1-q)-2(1-\gamma)\sqrt{(1-q)\gamma}}{(\gamma-q)} \leq p_2 < q$	γ	(B1) / (B2)
(5)	$\frac{1}{2} \leq \gamma < 1$, $1-\gamma \leq q < 1$:		
(5a)	$0 < p_2 \leq (1-q)(1-\gamma)$;	$\frac{(1-q)(1+\gamma)+p_2}{2}$	(A2)
(5b)	$(1-q)(1-\gamma) < p_2 < q\gamma$;	$p_2 + (1-q)\gamma$	(A1) / (A2)
(5c)	$q\gamma \leq p_2 < q$	γ	(B1) / (B2)

Following a similar manner, we can analyze the incumbent's best response corresponding to the demand case (B) in Table A1. Note that demand case (C) is irrelevant because $N_2 \equiv 0$ in this case, which means the entrant will not be able to make any profit if it prices within these regions. As a result, p_2 will not fall into this region in equilibrium, and hence the demand case (C) will not appear in equilibrium. Altogether, we summarize the incumbent's best response $p_1^*(p_2)$ in Table A2.

(3) We finally solve the entrant's optimal price p_2^*.

Based on the incumbent's best response functions in Table A2, we can formulate the entrant's profit function when anticipating the incumbent's best response in pricing, $\pi_2(p_2; p_1^*(p_2))$. For example, consider Case (1) in Table A2, that is, when $0 < \gamma < \frac{1}{2}$ and $0 < q \leq \gamma$.

$$\pi_2(p_2; p_1^*(p_2)) = \begin{cases} p_2\left(\frac{p_1^*(p_2)-p_2}{1-q} - \gamma\right) = p_2\frac{(1-q)(1-\gamma)-p_2}{2(1-q)}, & 0 < p_2 \leq q\gamma; \\ p_2\left(\frac{p_1^*(p_2)-p_2}{1-q} - \frac{p_2}{q}\right) = p_2\frac{q(1-q)-(2-q-\gamma)p_2}{2q(1-q)(1-\gamma)}, & q\gamma < p_2 < q\frac{(1-q)-\sqrt{(1-q)(1-\gamma)}}{\gamma-q}; \\ 0, & q\frac{(1-q)-\sqrt{(1-q)(1-\gamma)}}{\gamma-q} \leq p_2 < q. \end{cases} \quad (A.3)$$

It is easy to see that any $p_2 > q\frac{(1-q)-\sqrt{(1-q)(1-\gamma)}}{\gamma-q}$ cannot be optimal for the entrant. Therefore, we only need to focus on the first two segments of (A.3). The first order conditions for the first and the second segments yield $\hat{p}_2 = \frac{1}{2}(1-q)(1-\gamma)$ and $\hat{p}_2' = \frac{q(1-q)}{2(2-q-\gamma)}$, respectively. In order to determine the optimal p_2^*, we need to compare $\hat{p}_2$ and $\hat{p}_2'$ against the bounds 0, $q\gamma$, and $q\frac{(1-q)-\sqrt{(1-q)(1-\gamma)}}{\gamma-q}$. For example, if $0 < \gamma < \frac{1}{4}$, then $\hat{p}_2 > q\gamma$ and $q\gamma < \hat{p}_2' < q\frac{(1-q)-\sqrt{(1-q)(1-\gamma)}}{\gamma-q}$. As a result, $p_2^* = \hat{p}_2'$. In total, there are 6 subcases under Case (1). Table A3 shows the detailed parameter conditions with their respective equilibrium prices.

Following the same approach, we examine Cases (2) through (5) in Table A2 and derive the equilibrium prices within various sub-regions of parameter values, as we summarize in Table A3. Combining the cases with the same equilibrium outcome, we arrive at the equilibrium outcomes described in Proposition 1 and Table 1 (as we indicate in the rightmost column in Table A3). □

A.3 Proof of Corollary 1

Proof. Given that $\lim_{\gamma\uparrow 1}\frac{1-\gamma}{1+\gamma} = 0$, we can see that for any fixed $q < 1$, there exists a threshold $\gamma_1(q) \in (0,1)$ such that when $\gamma > \gamma_1(q)$, we enter region (iii). Then, it immediately follows that $\lim_{\gamma\uparrow 1}\pi_1^*(q,\gamma) = 1-q < \lim_{\gamma\uparrow 1}\pi_1^M(\gamma) = 1$. Consequently, there exists $\gamma_0(q) \in (\gamma_1(q), 1)$ such that for all $\gamma \in (\gamma_0(q), 1)$ we have $\pi_1^*(q,\gamma) < \pi_1^M(\gamma)$. □

Table A3: Optimal Pricing for the Entrant, p_2^*

Cases	Conditions	p_2^*	$p_1^*(p_2)$ Cases (as in Table A2)	Demand Cases (as in Table A1)	Equilibrium Cases (as in Table 1)
(1)	$0<\gamma<\frac{1}{2}$, $0<q\le\gamma$:				
	$0<\gamma\le\frac{1}{4}$, $0<q\le\gamma$;	$\frac{q(1-q)}{2(2-q-\gamma)}$	(1b)	(B3)	(i)
	$\frac{1}{4}<\gamma<\frac{2-\sqrt{2}}{2}$, $0<q\le\frac{2\gamma^2-4\gamma+1}{1-2\gamma}$;	$\frac{q(1-q)}{2(2-q-\gamma)}$	(1b)	(B3)	(i)
	$\frac{1}{4}<\gamma<\frac{2-\sqrt{2}}{2}$, $\frac{2\gamma^2-4\gamma+1}{1-2\gamma}<q\le\gamma$;	$q\gamma$	(1a) / (1b)	(A2) / (B3)	(ii)
	$\frac{2-\sqrt{2}}{2}\le\gamma\le\sqrt{2}-1$, $0<q\le\gamma$;	$q\gamma$	(1a) / (1b)	(A2) / (B3)	(ii)
	$\sqrt{2}-1<\gamma<\frac{1}{2}$, $0<q\le\frac{1-\gamma}{1+\gamma}$;	$q\gamma$	(1a) / (1b)	(A2) / (B3)	(ii)
	$\sqrt{2}-1<\gamma<\frac{1}{2}$, $\frac{1-\gamma}{1+\gamma}<q\le\gamma$	$\frac{1}{2}(1-q)(1-\gamma)$	(1a)	(A2)	(iii)
(2)	$0<\gamma<\frac{1}{2}$, $\gamma<q\le 1-\gamma$:				
	$0<\gamma<\frac{1}{4}$, $\gamma<q\le\frac{2\gamma^2-4\gamma+1}{1-2\gamma}$;	$\frac{q(1-q)}{2(2-q-\gamma)}$	(2b)	(B3)	(i)
	$0<\gamma<\frac{1}{4}$, $\frac{2\gamma^2-4\gamma+1}{1-2\gamma}<q\le\frac{1-\gamma}{1+\gamma}$;	$q\gamma$	(2a) / (2b)	(B3) / (A2)	(ii)
	$0<\gamma<\frac{1}{4}$, $\frac{1-\gamma}{1+\gamma}<q\le 1-\gamma$;	$\frac{1}{2}(1-q)(1-\gamma)$	(2a)	(A2)	(iii)
	$\frac{1}{4}\le\gamma<\sqrt{2}-1$, $\gamma<q\le\frac{1-\gamma}{1+\gamma}$;	$q\gamma$	(2a) / (2b)	(B3) / (A2)	(ii)
	$\frac{1}{4}\le\gamma<\sqrt{2}-1$, $\frac{1-\gamma}{1+\gamma}<q\le 1-\gamma$;	$\frac{1}{2}(1-q)(1-\gamma)$	(2a)	(A2)	(iii)
	$\sqrt{2}-1\le\gamma<\frac{1}{2}$, $\gamma<q\le 1-\gamma$	$\frac{1}{2}(1-q)(1-\gamma)$	(2a)	(A2)	(iii)
(3)	$0<\gamma<\frac{1}{2}$, $1-\gamma<q<1$	$\frac{1}{2}(1-q)(1-\gamma)$	(3a)	(A2)	(iii)
(4)	$\frac{1}{2}\le\gamma<1$, $0<q<1-\gamma$:				
	$\frac{1}{2}\le\gamma<1$, $0<q\le\frac{1-\gamma}{1+\gamma}$;	$q\gamma$	(4a) / (4b)	(A2) / (B3)	(ii)
	$\frac{1}{2}\le\gamma<1$, $\frac{1-\gamma}{1+\gamma}<q<1-\gamma$;	$\frac{1}{2}(1-q)(1-\gamma)$	(4a)	(A2)	(iii)
(5)	$\frac{1}{2}\le\gamma<1$, $1-\gamma\le q<1$	$\frac{1}{2}(1-q)(1-\gamma)$	(5a)	(A2)	(iii)

A.4 Proof of Proposition 2

Proof. According to Proposition 1, the entrant's equilibrium profit by choosing quality q can be written as

$$\pi_2(q)=\begin{cases}\frac{q(1-q)}{8(1-\gamma)(2-q-\gamma)}-cq, & 0<q\le\max\left\{\frac{2\gamma^2-4\gamma+1}{1-2\gamma},0\right\};\\ \frac{(1-\gamma-q)q\gamma}{2(1-q)}-cq, & \max\left\{\frac{2\gamma^2-4\gamma+1}{1-2\gamma},0\right\}<q\le\frac{1-\gamma}{1+\gamma};\\ \frac{1}{8}(1-q)(1-\gamma)^2-cq, & \frac{1-\gamma}{1+\gamma}<q<1.\end{cases} \tag{A.4}$$

Solving the first-order condition for the first segment of (A.4), we have $q_1^*=2-\gamma-\sqrt{\frac{(2-\gamma)(1-\gamma)}{1-8c(1-\gamma)}}$; the solution to the first-order condition for the second segment of (A.4) yields $q_2^*=1-\sqrt{\frac{\gamma^2}{\gamma-2c}}$. For the third segment of (A.4), because $\frac{d}{dq}\pi_2(q)=-\frac{1}{8}(1-\gamma)^2-c<0$, any $q>\frac{1-\gamma}{1+\gamma}$ cannot be optimal.

Define $\hat{\gamma}(c)$ as the unique solution to $-4\gamma^3 + 4(2c+3)\gamma^2 - 8(2c+1)\gamma + 8c + 1 = 0$ for $\gamma \in \left(0, \frac{2-\sqrt{2}}{2}\right)$. As we can verify, $\hat{\gamma}(c)$ is well defined for $c \in \left(0, \frac{\sqrt{2}-1}{4}\right)$. Note that $q_1^*(\hat{\gamma}) = q_2^*(\hat{\gamma}) = \frac{2\hat{\gamma}^2-4\hat{\gamma}+1}{1-2\hat{\gamma}}$.

Consider Case (a) when $0 < c \leq \frac{1}{16}$. We examine it in the following subcases.

(i) When $0 < \gamma \leq \hat{\gamma}(c)$, as we can show, $\frac{2\gamma^2-4\gamma+1}{1-2\gamma} > 0$, $0 < q_1^* \leq \frac{2\gamma^2-4\gamma+1}{1-2\gamma}$, and $q_2^* \leq \frac{2\gamma^2-4\gamma+1}{1-2\gamma}$. Therefore, $\pi_2(q)$ reaches its peak within the first segment of (A.4) at $q = q_1^*$, and any $q > \frac{2\gamma^2-4\gamma+1}{1-2\gamma}$ is suboptimal. As a result, the entrant's optimal quality choice is $q^* = q_1^* = 2 - \gamma - \sqrt{\frac{(2-\gamma)(1-\gamma)}{1-8c(1-\gamma)}}$.

(ii1) When $\hat{\gamma}(c) < \gamma < \frac{2-\sqrt{2}}{2}$, as we can show, $0 < \frac{2\gamma^2-4\gamma+1}{1-2\gamma} < q_1^*$ and $\frac{2\gamma^2-4\gamma+1}{1-2\gamma} < q_2^* < \frac{1-\gamma}{1+\gamma}$. Therefore, $\pi_2(q)$ reaches its peak within the second segment of (A.4) at $q = q_2^*$, and any $q < \frac{2\gamma^2-4\gamma+1}{1-2\gamma}$ or $q > \frac{1-\gamma}{1+\gamma}$ is suboptimal. As a result, $q^* = q_2^* = 1 - \sqrt{\frac{\gamma^2}{\gamma-2c}}$.

(ii2) When $\frac{2-\sqrt{2}}{2} \leq \gamma < \frac{1+\sqrt{1-8c}}{2}$, because $\frac{2\gamma^2-4\gamma+1}{1-2\gamma} \leq 0$, the first segment of (A.4) no longer applies. As we can show, $0 \leq q_2^* \leq \frac{1-\gamma}{1+\gamma}$ within this region. Therefore, $\pi_2(q)$ reaches its peak within the second segment of (A.4) at $q = q_2^*$, and any $q > \frac{1-\gamma}{1+\gamma}$ is suboptimal. As a result, $q^* = q_2^* = 1 - \sqrt{\frac{\gamma^2}{\gamma-2c}}$.

(iii) When $\frac{1+\sqrt{1-8c}}{2} \leq \gamma < 1$, $q_2^* < 0$. Therefore, $\pi_2(q)$ is decreasing in q for $q \in [0,1]$, and the optimal quality choice $q^* = 0$.

Combining (i) through (iii), we have the optimal quality choice of the entrant for Case (a) when $0 < c \leq \frac{1}{16}$. The other cases can be proven in a similar fashion. □

A.5 Proof of Corollary 2

Proof. Follows immediately from Proposition 2 by setting $q^*(\gamma, c) > 0$. □

A.6 Proof of Lemma 1

Proof. Comparing the equilibrium profits of the incumbent, π_1^*, in Proposition 1 and equation (2) over various parameter regions, we have:

(1) If $\gamma < \frac{1}{2}$, according to equation(2), the incumbent's monopoly profit $\pi_1^* = \frac{1}{4(1-\gamma)}$.

(i) In region (i) of Proposition 1, $\frac{(1-q)(4-q-3\gamma)^2}{16(1-\gamma)^2(2-q-\gamma)^2} > \frac{1}{4(1-\gamma)}$ if and only if $\frac{(\gamma-q)\left(4\gamma^2-(11-3q)\gamma+\left(q^2-5q+8\right)\right)}{16(1-\gamma)^2(2-q-\gamma)^2} > 0$. Note that $4\gamma^2-(11-3q)\gamma+\left(q^2-5q+8\right) > 0$ always holds because $(11-3q)^2-16\left(q^2-5q+8\right) = -7(1-q)^2 < 0$. Therefore, $\frac{(1-q)(4-q-3\gamma)^2}{16(1-\gamma)^2(2-q-\gamma)^2} > \frac{1}{4(1-\gamma)}$ if and only if $q < \gamma$.

(ii) In region (ii) of Proposition 1, $\frac{(1+\gamma-q)^2}{4(1-q)} > \frac{1}{4(1-\gamma)}$ if and only if $\frac{(\gamma-q)\left(-(1-\gamma)q+1-\gamma-\gamma^2\right)}{4(1-q)(1-\gamma)} > 0$. As we can show, because $q < \frac{1-\gamma}{1+\gamma}$ in this region, $\frac{(1+\gamma-q)^2}{4(1-q)} > \frac{1}{4(1-\gamma)}$ if and only if $q < \gamma$.

(iii) In region (iii) of Proposition 1, $\frac{1}{16}(1-q)(3+\gamma)^2 > \frac{1}{4(1-\gamma)}$ if and only if $q < 1 - \frac{4}{(1-\gamma)(3+\gamma)^2}$. Note that $\gamma = \frac{1-\gamma}{1+\gamma} = 1 - \frac{4}{(1-\gamma)(3+\gamma)^2}$ when $\gamma = \sqrt{2} - 1$.

(2) If $\gamma \geq \frac{1}{2}$, according to equation (2), the incumbent's monopoly profit $\pi_1^* = \gamma$.

(i) Region (i) of Proposition 1 does not apply when $\gamma \geq \frac{1}{2}$.

(ii) In region (ii) of Proposition 1, $\frac{(1+\gamma-q)^2}{4(1-q)} > \gamma$ if and only if $\frac{(1-q-\gamma)^2}{4(1-q)} > 0$, which always holds.

(iii) In region (iii) of Proposition 1, $\frac{1}{16}(1-q)(3+\gamma)^2 > \gamma$ if and only if $q < 1 - \frac{16\gamma}{(3+\gamma)^2}$. Note that $1 - \frac{16\gamma}{(3+\gamma)^2} = 1 - \frac{4}{(1-\gamma)(3+\gamma)^2}$ when $\gamma = \frac{1}{2}$.

Altogether, we can conclude that sharing its IP leads to higher equilibrium profit than remaining a monopoly when $q < \tilde{q}(\gamma)$, where $\tilde{q}(\gamma)$ is defined in (6) □

A.7 Proof of Proposition 3

Proof. First note that the equilibrium profit of the incumbent in a duopoly market, $\pi_1^*(q,\gamma)$ as derived in Proposition 1, is decreasing in the entrant's product quality q. To see this, we take the first derivative with respect to q. In region (i) of Table 1, $\frac{d}{dq}\pi_1^*(q,\gamma) = \frac{-(4-3\gamma-q)\left[(1-q)^2+3(1-\gamma)^2\right]}{16(1-\gamma)^2(2-q-\gamma)^3} < 0$; in region (ii), $\frac{d}{dq}\pi_1^*(q,\gamma) = -\frac{1}{4}\left[1 - \frac{\gamma^2}{(1-q)^2}\right] < 0$ because $q < \frac{1-\gamma}{1+\gamma} < 1-\gamma$ in this region; in region (iii), $\frac{d}{dq}\pi_1^*(q,\gamma) = -\frac{1}{16}(3+\gamma)^2 < 0$.

We prove the results summarized in Table 3 by deviding the value range of γ into five cases: (a) $0 < \gamma \leq \frac{1}{4}$; (b) $\frac{1}{4} < \gamma \leq \frac{2-\sqrt{2}}{2}$; (c) $\frac{2-\sqrt{2}}{2} < \gamma \leq \tilde{\gamma}$; (d) $\tilde{\gamma} < \gamma \leq \frac{3-\sqrt{5}}{2}$; (e) $\frac{3-\sqrt{5}}{2} < \gamma < 1$. We analyze case (a) ($0 < \gamma \leq \frac{1}{4}$) in detail below. The other cases can be analyzed in a similar fashion.

(1) Recall the analysis and results for Proposition 2 and Lemma 1. For $0 < \gamma \leq \frac{1}{4}$, $q^*(\gamma, c) = 2 - \gamma - \sqrt{\frac{(2-\gamma)(1-\gamma)}{1-8c(1-\gamma)}}$, and $\tilde{q}(\gamma) = \gamma$. Solving $q^*(\gamma,c) = \tilde{q}(\gamma)$ for c, we have $c = \frac{2-3\gamma}{32(1-\gamma)^2}$. Recall that $q^*(\gamma,c)$ is decreasing in c. Therefore, if $c < \frac{2-3\gamma}{32(1-\gamma)^2}$, $q^*(\gamma,c) > \tilde{q}(\gamma)$, so sharing its IP would lead to less profit for the incumbent than not sharing and remaining a monopoly. If $k > \frac{32(1-\gamma)^2}{2-3\gamma}$, with either basic sharing (i.e., $\rho = 1$) or advanced sharing (i.e., $\rho = 2$), $c = \frac{1}{\rho k} < \frac{2-3\gamma}{32(1-\gamma)^2}$. As a result, either basic or advanced sharing is dominated by remaining a monopoly, so the optimal strategy for the incumbent is no sharing (i.e., $\rho^* = 0$). Thus, this case constitutes a part of region (1) in Table 3.

(2) Solving $q^*(\gamma,c) = 2-\gamma-\sqrt{\frac{(2-\gamma)(1-\gamma)}{1-8c(1-\gamma)}} = 0$ for c, we have $c = \frac{1}{8(2-\gamma)(1-\gamma)}$. Therefore, if $\frac{2-3\gamma}{32(1-\gamma)^2} < c < \frac{1}{8(2-\gamma)(1-\gamma)}$ (note that $\frac{2-3\gamma}{32(1-\gamma)^2} < \frac{1}{8(2-\gamma)(1-\gamma)}$ for $\forall\gamma \in \left(0,\frac{1}{4}\right)$), $0 < q^*(\gamma,c) < \tilde{q}(\gamma)$, so sharing its IP leads to more profit for the incumbent than not sharing, and meanwhile, the entrant is willing to enter the market by achieving a strictly positive level of net profit with $q^* > 0$. For this reason, if $8(2-\gamma)(1-\gamma) < k < \frac{32(1-\gamma)^2}{2-3\gamma}$, the optimal strategy for the incumbent is basic sharing (i.e., $\rho^* = 1$ so that $c = \frac{1}{k}$). Note that advanced sharing (i.e., $\rho = 2$ so that $c = \frac{1}{2k}$) cannot be optimal in this case because as we have shown, $\pi_1^*(q,\gamma)$ (as in Proposition 1) is decreasing in q, so helping the entrant reduce development cost (which would increase $q^*(\gamma,c)$) would hurt the incumbent's equilibrium profit. Altogether, this case constitutes a part of region (2) in Table 3.

(3) When $\frac{16(1-\gamma)^2}{2-3\gamma} < k < 8(2-\gamma)(1-\gamma)$ (note that $\frac{16(1-\gamma)^2}{2-3\gamma} < 8(2-\gamma)(1-\gamma)$ for $\forall\gamma \in \left(0,\frac{1}{4}\right)$), if the incumbent chooses basic sharing (i.e., $\rho = 1$), then $c = \frac{1}{k} > \frac{1}{8(2-\gamma)(1-\gamma)}$. By the above analysis in (2), $q^*(\gamma,c) = 0$. In other words, with basic sharing, the entrant would be unable to achieve a positive level of net profit, and hence $q^* = 0$ would be its optimal quality choice; as a result, the entrant would not enter the market in the first place. If the incumbent chooses advanced sharing (i.e., $\rho = 2$), then $c = \frac{1}{2k} < \frac{2-3\gamma}{32(1-\gamma)^2}\left(< \frac{1}{8(2-\gamma)(1-\gamma)}\right)$. By the above analysis in (1), $q^*(\gamma,c) > \tilde{q}(\gamma)\,(> 0)$, indicating that the incumbent could achieve more profit by remaining a monopoly than advanced sharing. Altogether, neither basic nor advanced sharing can outperform the monopoly profit. Therefore, the optimal strategy for the incumbent is no sharing (i.e., $\rho^* = 0$). This case hence constitutes a part of region (3) in Table 3.

(4) When $4(2-\gamma)(1-\gamma) < k < \frac{16(1-\gamma)^2}{2-3\gamma}\,(< 8(2-\gamma)(1-\gamma))$, if the incumbent chooses basic sharing (i.e., $\rho = 1$), then $c = \frac{1}{k} > \frac{1}{8(2-\gamma)(1-\gamma)}$. By the above analysis in (2), $q^*(\gamma,c) = 0$. As a result, the entrant would not enter the market in the first place, and the incumbent would maintain the monopoly profit. If the incumbent chooses advanced sharing (i.e., $\rho = 2$), then $c = \frac{1}{2k}$, so $\frac{2-3\gamma}{32(1-\gamma)^2} < c < \frac{1}{8(2-\gamma)(1-\gamma)}$. By the above analysis in (1) and (2), $0 < q^*(\gamma,c) < \tilde{q}(\gamma)$. As a result, the entrant is willing to enter the market by achieving a strictly positive level of net profit with $q^* > 0$, and meanwhile, the incumbent can achieve more profit than remaining a monopoly. Therefore, the optimal strategy for the incumbent is advanced sharing (i.e., $\rho^* = 2$). This case hence constitutes a part of region (4) in Table 3.

(5) When $k < 4(2-\gamma)(1-\gamma)$, even with advanced sharing (i.e., $\rho = 2$), $c = \frac{1}{2k} > \frac{1}{8(2-\gamma)(1-\gamma)}$,

so $q^* (\gamma, c) = 0$. In other words, even with advanced sharing, the entrant would still be unable to achieve a positive level of net profit, and hence would not enter the market in the first place. As a result, the incumbent remains a monopoly in equilibrium. This case hence constitutes a part of region (5) in Table 3. $\square$