Oligopolistic Pricing with Online Search

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ABSTRACT: In this paper, we set up a game-theoretic model to examine oligopolistic price competition, considering two features of online search: the existence of a common search ordering and shoppers who have nonpositive search cost. We find that in equilibrium firms set their prices probabilistically rather than deterministically, and different firms follow different price distributions. The equilibrium pricing pattern exhibits an interesting *local-competition* feature in which direct price competition occurs only between firms adjacent to each other. Further, we incorporate consumers' search strategies into the model so that both search order and stopping rules are

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determined rationally by consumers. We show that similar patterns may continue to hold in the fully rational framework when consumers have higher inspection costs for inferior positions.

THE INTERNET HAS GREATLY IMPROVED THE EFFICIENCY of sharing information. On the one hand, the Internet greatly reduces the physical cost of accessing product information; on the other hand, the rapidly developed e-commerce applications, especially online sponsored advertising, brings merchants selling similar products together and facilitates consumers' searching. Nevertheless, the prediction of "the law of one price" has not been realized in the Internet era. Price dispersion has been well documented and discussed in the information systems literature [5, 7, 17]. This work draws upon consumer online search behavior and studies the oligopolistic equilibrium pricing to offer a theoretical explanation of online price dispersion.

One classical view in economic theory attributes price dispersion to the heterogeneity in consumers' search behavior [3, 18]. These works typically study price competition in traditional offline search markets, and their model settings may not fit the online search very well. Compared to the traditional offline search, two features are more salient in the online environment: first, because of considerable differences in the location or the visibility of business positions (e.g., the hyperlinks), there often exists a common ordering in consumer search. Second, consumers' search costs are highly diversified; in particular, there exists a substantial portion of "shoppers" who have nonpositive search costs.

Differences in location and the way of presenting on a Web page, or different virtual positions in the hyperlinking network on the Internet, cause firms' links to differ greatly in terms of their visibility or prominence level. For example, search engines commonly display a limited number of premium-sponsored links in a highlighted area with a large size and a bright color on the top of a search result page; they also display a list of regular-sponsored links in the right column of a Web page with no highlighting, among which the top positions are commonly believed to be superior to the lower positions. When display space is limited (e.g., on mobile devices), sometimes only one or two ads are displayed at a time, and users have to switch to another page to view additional ads. Because of human eye movement patterns and information-processing habits, consumers usually pay different levels of attention to different positions. Experimental studies have shown that consumers pay more attention to content with colors and a large size [13], and compared to paper media, consumers are more likely to focus on ads near the heading of electronic lists [10]. Online statistics also show a significant decrease in traffic from the top-sponsored link downward and much fewer visits after the first page [2]. As a result, most consumers browse links following a common order: they inspect the most prominent positions first, and then some stop while others continue to inspect the less prominent ones. In fact, there are also examples in the physical world that resemble

such ordering: most consumers first look at shelf slots at the eye level in a supermarket [9] or first visit the storefronts near the main entrance of a shopping mall, and then some of them continue to the floor-level shelf slots or the corner storefronts.

In reality such a common ordering could simply be the direct result of human habits and not necessarily a strategic decision after sophisticated calculation;[1] nevertheless, we can also explain such ordering in a fully rational framework. As we show, the ordered search can be derived as an equilibrium outcome originating from differences in the inspection costs for different positions. In particular, the inferior positions incur higher inspection costs than the superior ones. A higher inspection cost can be interpreted as the psychological resistance to overcoming the information-processing habits, or the extra effort in locating a less-prominent link or switching Web pages. Given such differences, consumers' choice to follow a certain order is a rational search strategy in equilibrium.

The second feature of online search behavior owes to the convenience brought by the Internet, which reduces consumers' search costs from driving to the store to making several clicks of the mouse. In addition, it has also been shown that some consumers derive hedonic utility from shopping online [6]. As a result, a few consumers actually have a nonpositive (zero or even negative) net search cost. We call them *shoppers*. However, not everyone shopping online has such time luxury. The convenience of electronic commerce attracts people with time constraints whose only goal is to find a product within the shortest amount of time. Thus, these consumers have positive search costs. They do not conduct an exhaustive search and stop searching at certain stages, usually after sampling only a few sites (e.g., [11] empirically shows that online customers tend to search very few sites on average).

To study the equilibrium pricing pattern, we set up a game-theoretic model capturing the two features of the online search. We consider oligopolistic competition in which multiple firms compete for consumers in a product market. Firms are differentiated in the prominence level of their positions, which are reflected in the ranks in consumers' presumed search sequence. Consumers are differentiated in their search behavior. In particular, a few consumers have nonpositive search costs and conduct a thorough search. We eliminate heterogeneity among firms and consumers in all other dimensions except firms' position and consumers' search behavior to show that the driving forces of the equilibrium pricing pattern are the two distinctive features of the search behavior.

We find an interesting equilibrium pricing pattern when first taking consumers' search strategies as exogenous. The equilibrium exhibits the feature of "local competition," in which firms compete directly with their neighbors along consumers' search order only. We show that in equilibrium all firms mix their prices over different supports. Overlaps occur only in the supports of two firms adjacent to each other. Hence, there is no direct competition between any two firms that are not next to each other. We further show that behind the local-competition feature lies a global mutual dependence across all firms. We then endogenize consumers' search strategies so that consumers make fully rational decisions on their search order and stopping rules. We show that similar pricing patterns may continue to hold in equilibrium when consumers' inspection costs for different positions are different.

The main contribution of this work to the economic theory on search and pricing lies in that we study the asymmetric mixed-strategy equilibrium pricing in oligopolistic competition. To the best of our knowledge, the local-competition pattern revealed in this study is absent in the previous literature. The investigation on endogenizing the consumer search enriches the studies that explore asymmetric equilibrium pricing with optimal search strategies.

Diamond [8] raises the famous paradox that when consumers have positive search costs, an endogenous search model leads to a trivial equilibrium in which all firms charge the monopoly price and consumers do not search. Varian [20] suggests that when there exist consumers who are "informed" of all firms' prices, the equilibrium outcome may involve mixed-strategy pricing, which leads to price dispersion. Stahl [18] studies a random search model in which consumers randomly pick a firm to inspect and all firms are symmetric. That article considers the existence of a portion of "shoppers" who have zero search cost and derive a symmetric equilibrium pricing provided that all nonshoppers have the same search cost. Weitzman [22] formulates the optimal search strategies given firms' (asymmetric) price distributions in a general setting. Arbatskaya [1] is among the few who study ordered search and price competition. By considering cost distributions atomless at the zero point, that article focuses on the pure-strategy equilibrium. There are other studies that analyze mixed-strategy equilibrium in duopolistic competition in different contexts. For example, Campbell et al. [4] consider the effect of shoppers in a symmetric duopolistic setting, and Narasimhan [14], Weber and Zheng [21], and Xu et al. [23] consider asymmetric duopolistic mixed strategies. In contrast to these works, we consider the existence of shoppers in the ordered search market, and explicitly derive asymmetric mixed-strategy equilibrium in oligopolistic price competition with both exogenous and endogenous consumer search.

The rest of the paper is organized as follows. Next, we start with a model in which consumers' search behavior is exogenously given. The third section details the analysis and shows the main results. In the fourth section, we endogenize the consumer search such that consumers rationally decide what search order to follow and when to stop. We consider both cases of position-invariant and position-dependent inspection costs. We show that the local-competition pattern may arise in the fully rational framework when consumers have higher inspection costs for inferior positions. The paper concludes with a discussion on managerial implications.

Model

THERE ARE n (≥ 2) FIRMS SELLING HOMOGENEOUS PRODUCTS and competing for consumers in a product market. These firms have the same marginal production cost, which is normalized to zero without loss of generality. A continuum of consumers with unit mass exists in the market. Each consumer has a unit demand of the product and realizes a unit utility from consuming the product. Therefore, consumers will buy the product only if its price does not exceed 1. Firms are identical except for their rank

in the search ordering, and consumers are identical except for their search behavior. By eliminating differentiation in all other dimensions, we are able to show that the distinctive features of consumers' online search behavior alone could drive an interesting price dispersion pattern.

Consumers obtain product information through an information portal with a list of hyperlinks directed to firms' Web sites, where purchases can be made directly. Firms are placed in different positions in the list, which can be viewed as the outcome of a pregame competition, such as a bidding competition. Because all firms are identical ex ante, the location competition outcome is irrelevant for analyzing the price competition. Any assignment of positions becomes identical after relabeling firms by their position rank. Therefore, we do not include the location competition in the model but start from *after* firms get placed at different positions. Different positions have different prominence levels, which can be strictly ordered. Without loss of generality, we call the most prominent position the *first* position, the second most prominent position the *second* position, and so on. For convenience, we call the firm at the ith position *firm i* ($i = 1, ..., n$).

We start with the case in which consumers' search strategies are exogenously given, in a way reflecting the two unique features of online search patterns: first, there exists a common search ordering so that all consumers start searching from the first position and may continue to the second, then the third, and so forth. Second, consumers' search costs are highly diversified so that they may stop searching at different stages. In particular, there exists a certain portion of *shoppers* who have a nonpositive search cost and sample all positions before making the purchase decision. Specifically, we assume that after sampling the ith position, a portion of α_i ($0 < \alpha_i < 1$) stop searching, while the other $1 - \alpha_i$ continue to sample the next position. Therefore, the portion of consumers who visit the ith ($i \geq 2$) position is $\Pi_{j=1}^{i-1}(1 - \alpha_j)$. To simplify notation, we denote $\beta_i = \Pi_{j=1}^{i-1}(1 - \alpha_j)$, and let $\beta_1 = 1$ and $\alpha_0 = 0$. To rule out violent fluctuation in the attention-declining rates α_is, we assume that $\alpha_i \geq \alpha_{i+1}(1 - \alpha_i)$ ($1 \leq i < n$). This condition requires that the rate of decline in attention (i.e., α_is) does not increase dramatically from one position to the next, which can be easily satisfied (e.g., the same rate of decline across positions ($\alpha_i = \alpha_{i+1}$) satisfies this condition).[2]

The timing of the game is as follows. Firms at different positions price their products simultaneously. Consumers sample the position(s), learn the price(s), and make the purchase decision. For those who sample at least two positions, they purchase from the firm with the lowest price. When there is a tie in the lowest price, they randomly pick one of the firms, with equal probability. A brief summary of notation is presented in Appendix A.

Equilibrium with Exogenous Search

WE FIRST DERIVE FIRMS' EQUILIBRIUM PRICING STRATEGIES and then analyze the pattern of equilibrium price dispersion. In deriving the equilibrium pricing, we notice that any static pricing is unstable due to the existence of shoppers.

Lemma 1 (Lack of Pure-Strategy Equilibrium): There is no pure-strategy equi-librium in the price competition.

Proof: We prove the above result by contradiction. Suppose there is a pure-strategy pricing equilibrium. If there is no tie in the lowest price, then it is profitable for the firm with the lowest price to deviate by increasing its price closer to but still lower than the second-lowest price. If there is a tie in the lowest price that is strictly positive, then any of the firms with the lowest price has profitable devia-tion by slightly cutting the price. If there is a tie in the lowest price that is zero, then among all these firms with zero price, the one with the highest prominence level can achieve positive profit by slightly increasing its price. Therefore, pure-strategy pricing cannot be an equilibrium. Q.E.D.

Because there exists a certain portion of consumers who sample all positions to look for the lowest price, a slight cut in price to offer the lowest can lead to a significant increase in market share by capturing this portion of consumers. As a result, compet-ing firms tend to lower their prices relative to the rivals. However, once the price is pushed to a certain low level, the firm at a better position in terms of prominence can be better off by charging a higher price and exploiting those consumers who stop searching right there. Therefore, any pricing strategy in which firms statically charge one price cannot be stable. Clearly, the driving force here is the presence of shoppers and the locational asymmetry created by the search ordering.

We next examine the mixed-strategy pricing equilibrium. We use $F_i(p)$, $i = 1, ..., n$, to describe firm i's mixed strategy of pricing. Like regular cumulative distribution functions, $F_i(p)$ measures the probability that firm i charges a price less than or equal to p.

Proposition 1 (Equilibrium Pricing and Local-Competition Pattern): The equi-librium mixed strategy of pricing from position i is as follows:

$$F_i(p) = \begin{cases} 1 - \dfrac{\bar{p}_{i+1}}{p} & p \in [\bar{p}_{i+1}, \bar{p}_i) \\ 1 - \dfrac{\alpha_{i-1}(\bar{p}_{i-1} - p)}{\alpha_i(1-\alpha_{i-1})p} & p \in [\bar{p}_i, \bar{p}_{i-1}] \end{cases} \quad (1 \le i \le n-1)$$

$$(1)$$

$$F_n(p) = 1 - \frac{\alpha_{n-1}(\bar{p}_{n-1} - p)}{(1-\alpha_{n-1})p} \quad p \in [\bar{p}_n, \bar{p}_{n-1}],$$

where $\bar{p}_i s$ are recursively defined as

$$\bar{p}_0 = \bar{p}_1 = 1$$
$$\bar{p}_i = k_{i-1}\bar{p}_{i-1} \quad (i = 2, ..., n),$$

$$(2)$$

and the coefficient $k_i s$ are recursively defined as

$$k_{n-1} = \alpha_{n-1}$$
$$k_i = \frac{\alpha_i}{\alpha_i + \alpha_{i+1}(1-\alpha_i)k_{i+1}} \quad (i = n-2, ..., 1).$$

$$(3)$$

Proof: All proofs are presented in Appendix B, unless indicated otherwise.

We next use an example to illustrate the pattern of the equilibrium mixed-strategy pricing.

> Example 1: Consider a case of four positions with the same declining rate. Specifically, $n = 4$ and $\alpha_1 = \alpha_2 = \alpha_3 = 1/2$. According to the recursive definition in Equation (3), $k_3 = \alpha_3 = 1/2$, and we can then derive that $k_2 = 4/5$ and, further, that $k_1 = 5/7$. Thus, according to Equation (2), $\bar{p}_1 = 1$, $\bar{p}_2 = k_1\bar{p}_1 = 5/7$, $\bar{p}_3 = k_2\bar{p}_2 = 4/7$, and $\bar{p}_4 = k_3\bar{p}_3 = 2/7$. Notice that by definition, the sequence of price support bounds $\{\bar{p}_i\}_{i=1}^n$ is monotonically decreasing, so that $\bar{p}_1 > \bar{p}_2 > \bar{p}_3 > \bar{p}_4$. The pricing strategies of the four firms are as follows:

$$
F_1(p) = \begin{cases} 1 - \dfrac{5}{7p} & p \in \left[\dfrac{5}{7}, 1\right) \\ 1 & p = 1 \end{cases}
$$

$$
F_2(p) = \begin{cases} 1 - \dfrac{4}{7p} & p \in \left[\dfrac{4}{7}, \dfrac{5}{7}\right) \\ 1 - \dfrac{2(1-p)}{p} & p \in \left[\dfrac{5}{7}, 1\right] \end{cases}
$$

$$
F_3(p) = \begin{cases} 1 - \dfrac{2}{7p} & p \in \left[\dfrac{2}{7}, \dfrac{4}{7}\right) \\ 1 - \dfrac{2(5-7p)}{7p} & p \in \left[\dfrac{4}{7}, \dfrac{5}{7}\right] \end{cases}
$$

$$
F_4(p) = 1 - \dfrac{4-7p}{7p} \qquad p \in \left[\dfrac{2}{7}, \dfrac{4}{7}\right].
$$

(4)

Figure 1 illustrates the supports and distributions for the pricing strategies of the firms in these four positions. Figure 2 depicts the simulated results of the equilibrium in Example 1 with each point representing an independent draw from the price distributions in Equation (4). Notice that the dotted square areas that the diagonal passes through in (a), (b), and (d) indicate the direct price competition between two adjacent firms.

In equilibrium, each firm achieves a constant expected payoff by charging any price within its price support. On the one hand, the constant payoff makes each firm willing to randomize price over its entire support. On the other hand, each firm randomizes its price in a way that gives its competitors a constant payoff within their price supports. Specifically, firm i's expected profit can be written as

$$
\begin{cases} \pi_i(p) = p\alpha_i\beta_i + p\alpha_{i+1}\beta_{i+1}\left[1 - F_{i+1}(p)\right] & p \in \left[\bar{p}_{i+1}, \bar{p}_i\right) \\ \pi_i(p) = p\alpha_i\beta_i\left[1 - F_{i-1}(p)\right] & p \in \left[\bar{p}_i, \bar{p}_{i-1}\right]. \end{cases}
$$

(5)

When firm i ($2 \le i \le n - 2$) charges a price within the lower half of its price support $[\bar{p}_{i+1}, \bar{p}_i)$, the firm captures all consumers who sample its site and stop searching there (i.e., a portion of $\alpha_i\beta_i$). This is because its price is lower than firm $(i - 1)$'s price for
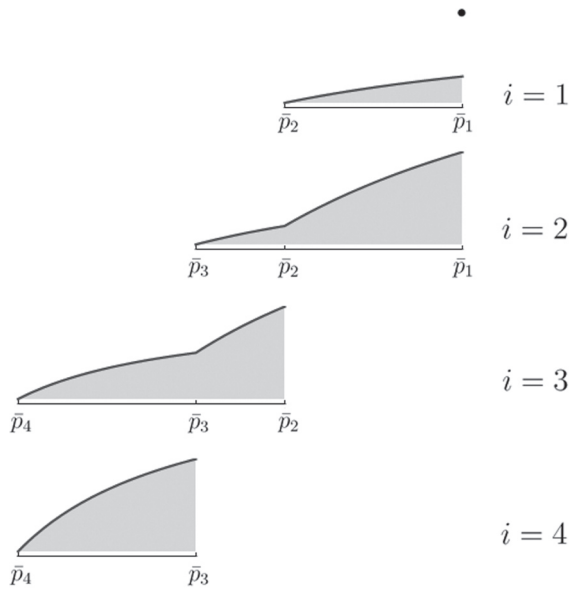
Figure 1. Price Supports and Cumulative Distributions for Different Positions

sure (recall that firm $(i-1)$'s price support is $[\bar{p}_i, \bar{p}_{i-2}]$), which accounts for the first term of the right-hand side of the first equation in Equation (5). Firm i can capture those who continue to sample the $(i+1)$th position and stop searching there (i.e., a portion of $\alpha_{i+1}\beta_{i+1}$) only when its price is lower than firm $(i+1)$'s price, which accounts for the second term. Naturally, firm i forgoes all those consumers who continue to sample the $(i+2)$th position, since firm $(i+2)$'s price support is $[\bar{p}_{i+3}, \bar{p}_{i+1}]$ and its price is lower for sure. The second equation in Equation (5) is the expected profit when firm i prices within the upper half of its price support, which can be interpreted in a similar way. Substituting in the firms' equilibrium pricing strategies and by simple algebra, we can show that by charging any price within its support, firm i achieves a constant expected profit $\alpha_i\beta_i\bar{p}_i$.

The most interesting feature of the equilibrium pricing is the *local-competition* pattern; that is, firms compete directly with their neighbors only. There is no overlap between the price supports of any two firms more than one position apart and thus no direct price competition between firms "distant" from each other. The driving forces of such a pattern are the two features of consumers' search behavior. Because of the existence of shoppers, firms do not statically charge one single price but have to mix their prices to compete for consumers. Nevertheless, such competition is localized because of the decrease in visits along the common search order. The firm at a lower position cannot be better off by entering a higher price range, because it would then lose its already quite limited customer base. The firm at a higher position will not undercut its price, because in doing so it would only entangle itself in a fiercer price competition against the lower-ranked firms, which would result in little gain in extra
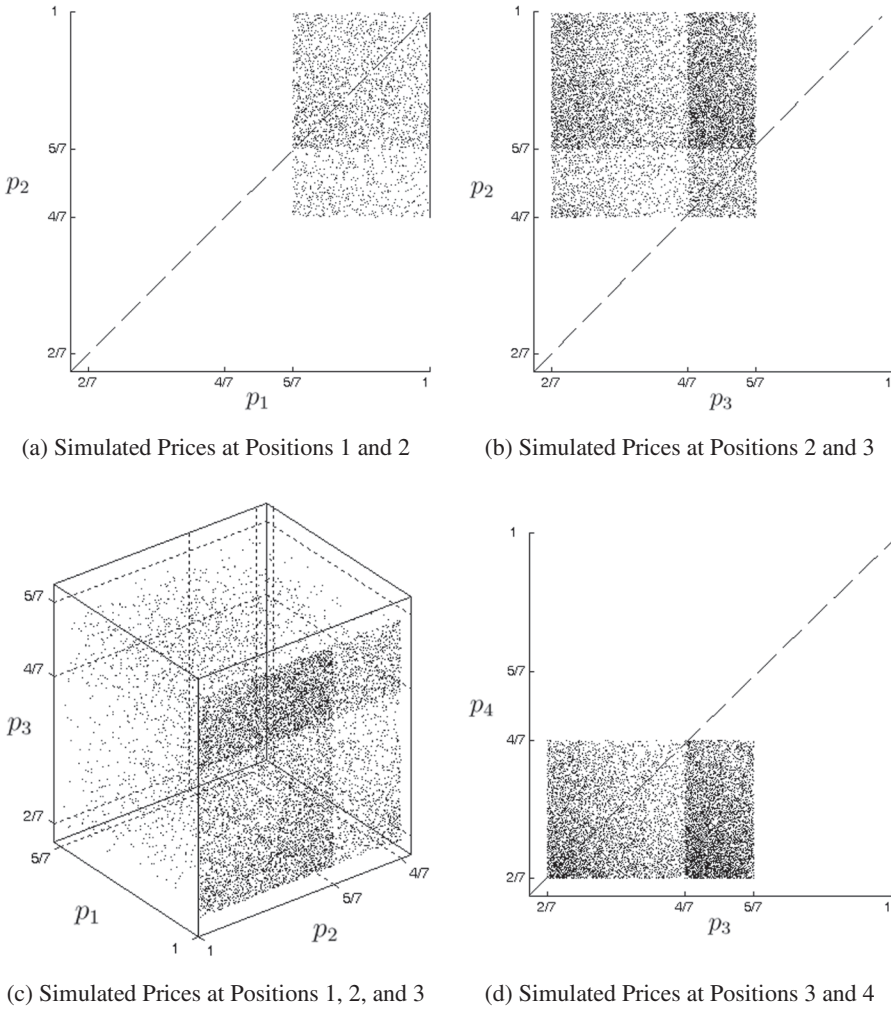
(a) Simulated Prices at Positions 1 and 2

(b) Simulated Prices at Positions 2 and 3

(c) Simulated Prices at Positions 1, 2, and 3

(d) Simulated Prices at Positions 3 and 4

Figure 2. Simulation of the Equilibrium in Example 1 (10,000 Simulated Points)

demand but significant loss in the profit from the captured demand. As a result, to compete locally is the equilibrium outcome.

Another interesting aspect of the local-competition pattern is that each firm's equilibrium pricing strategy only involves "local" information. According to Equation (1), firm i's price distribution F_i only contains consumer search parameters α_{i-1}, α_i, and α_{i+1}. Also, firm i's price support is determined by (the lower bound of) firm $(i-2)$'s support and (the upper bound of) firm $(i+2)$'s support; within its support, firm i's profit is determined only by firm $(i+1)$'s and firm $(i-1)$'s pricing strategies, according to Equation (5). In this sense, although the formal equilibrium analysis needs to be based on the whole picture of the game, to formulate optimal pricing strategy in practice, decision makers can simply focus on the traffic information at the adjacent

positions and the pricing strategies of the neighboring firms (i.e., firms that are adjacent and one position apart).

It is worth noting that the local-competition pattern is the result of global consideration. Although no direct price competition is explicitly observed in equilibrium, even firms distantly above or below have an impact on a particular firm's pricing strategies. In fact, although firm $i + k$ ($i − k$) does not compete directly with firm i, it affects firm i's pricing through a chain effect, from firm $i + k − 1$ ($i − k + 1$) through firm $i + 1$ ($i − 1$). To see this, reconsider Example 1. When the fourth firm is eliminated, for example, although the first two firms still have the same neighbors as before, the price support of the first firm shifts toward the right from $[5/7, 1]$ to $[4/5, 1]$, and the second firm's support shifts to the left from $[4/7, 1]$ to $[2/5, 1]$. In fact, when the competitor from below disappears, the third firm tends to increase its price (and shifts its price support from $[2/7, 5/7]$ to $[2/5, 4/5]$). In response, the second firm lowers its price support to capture more demand. Meanwhile, the more intense competition between the second and the third firms drives away the first firm's interest, which leads to the increase of the lower bound of its price support. As we can see, behind the local-competition phenomenon actually lies the global mutual dependence among all firms.

Several other features of equilibrium pricing are also worth noting. First, except for the first one, all firms' equilibrium pricing strategies are atomless within their entire supports, including the upper and lower bounds. This is because a mass point in one firm's price distribution would result in a downward jump of another firm's expected demand at that point and, consequently, lower profit levels in a contiguous region right to that point. For this reason, the only possible place where a mass point may occur is the common upper bound of the first two firms' price supports $\bar{p}_1$: although the mass point in $F_1(\cdot)$ causes a downward jump in firm 2's expected profit at $p = \bar{p}_1$, firm 2's actual expected profit is not affected because $F_2(\cdot)$ places a nonpositive probability measure on that particular point. This feature under our oligopolistic model is in line with the results derived from duopolistic competition in other settings [12].

The kinks in firms' pricing distributions can be explained by the localized competition. The pricing distributions in the first and second parts of the support for firm i ($2 \leq i \leq n − 1$) are determined by the competition against its direct neighbors, firms $i − 1$ and $i + 1$, respectively. As the competition against the firm above and the firm below are generally different, naturally a kink arises in firm i's pricing distribution at $\bar{p}_i$. The shape of pricing distributions for the first and last firms is distinctive from the rest because those two firms have one direct neighbor only.

The next corollary reveals the monotonic decrease of the expected profit of firms at different positions, which explains why the top position of a sponsored list in online search advertising is usually the most popular and engenders fierce bidding competition.

Corollary 1 (Decrease of Expected Profits): The firms' equilibrium expected profits decrease monotonically from the first toward the last; that is, $\pi_i > \pi_{i+1}$, $i = 1, ..., n − 1$.

Proof: Note that $\pi_i = \alpha_i \beta_i \bar{p}_i$. Because $\bar{p}_i > \bar{p}_{i+1}$ and $\alpha_i \geq \alpha_{i+1}(1 − \alpha_i)$, $\pi_i > \pi_{i+1}$. Q.E.D.

Corollary 1 shows that location advantage is rewarding in the sense that the firm in the advantageous location earns a higher profit. It is worth pointing out that the profit difference between a higher-ranked position and a lower-ranked one should dissipate in the pregame location competition if all firms are ex ante identical. That is, a firm has to pay a higher price for a superior position, which counterbalances its profit advantage. There is a rich literature on the competition for better locations or exposure, from the classical advertising literature [16] to the recent work on online advertising and position auctions [21].

The next corollary indicates that the equilibrium pricing under the ordered search with shoppers exhibits two levels of dispersion: not only are the realized prices at different positions different, but the expected prices are also different across positions.

> *Corollary 2 (Decrease of Expected Prices): The expected price decreases monotonically from the first position toward the last one; that is, $E(p_i) > E(p_{i+1})$, $i = 1, ..., n - 1$.*

Notice that firms adjacent to each other adopt similar pricing strategies over the overlapped interval of their supports. In fact, according to Equation (4), the conditional probability density functions of their pricing strategies over the overlapped interval are the same, which implies the same conditional expectations; that is, $E(p_i | p \in [\bar{p}_{i+1}, \bar{p}_i]) = E(p_{i+1} | p \in [\bar{p}_{i+1}, \bar{p}_i])$. Because firm i prices within the upper half of its price support $[\bar{p}_i, \bar{p}_{i-1}]$ with positive probability and firm $i + 1$ prices within the lower half of the support $[\bar{p}_{i+2}, \bar{p}_{i+1}]$ with positive probability, the unconditional expectation of firm i's price is strictly higher than that of firm $(i + 1)$.

Corollary 2 shows that the equilibrium price expectation decreases monotonically along the direction of consumers' search ordering. As a result, search is rewarding in the sense that those who keep searching are more likely to find a lower price.

Equilibrium with Endogenous Search

IN THE PREVIOUS ANALYSIS, WE TAKE CONSUMERS' SEARCH BEHAVIOR, including the search order and stopping rules, as exogenously given. In this section, we extend the analysis to endogenize consumers' search strategies. The focus is to explore whether and under what conditions a similar equilibrium pricing pattern continues to arise in the fully rational framework. As we show, the inherent difference among positions is necessary for the local-competition pattern to arise in equilibrium pricing. When consumers are free to sample any position with no particular ordering constraint and inspecting different positions incurs the same cost, such a pattern disappears. The pattern arises only if the more prominent position incurs a lower sampling cost than the less prominent position, which reflects the underlying distinction between ordered search and random search. In this case, we further show that under certain parametric conditions, equilibrium pricing with the same local-competition pattern can be derived in the fully rational framework.

We now consider consumers as active players in the game. We assume that all consumers are fully rational and decide their search order and stopping rules strategically. We consider the *rational-expectations equilibrium (REE),* in which consumers' search

strategies are rational given firms' pricing strategies and firms have no profitable deviation in pricing given consumers' search strategies. For shoppers with zero search cost, it is always optimal for them to sample all positions before making a purchase decision. For nonshoppers, we consider a sequential search process: the consumer inspects one position and learns the price, and then he or she decides whether to continue searching or to stop and, if to continue, which position to inspect next. In other words, an individual consumer's search strategy consists of a sequence of decisions; each decision $d(z, C)$ can be to stop, to inspect a position, or to randomly inspect several positions with a probability distribution; $d(z, C)$ depends on the lowest price z from all the inspected positions, and the choice set C, which contains all the uninspected positions. To determine $d(z, C)$, the consumer needs to calculate the net expected gain from all possibilities for the next step. The net expected gain from inspecting the ith position, given the lowest sampled price z and the choice set C, $EG(i; z, C)$, equals the expected decrease in purchase price plus the net expected gain from another rational search afterward, minus the inspection cost. Notice that when no position has been inspected yet, we let $z = 1$, and we thus only need to consider $z \leq 1$ throughout the rest of the paper. Formally, for an individual consumer with positive inspection costs k_i, given firms' pricing strategies F_i, similar to the formation in Weitzman [22], the net expected gain can be formulated recursively as

$$EG(i; z, C) = \int_{\underline{p}}^{z}(z - p)\, dF_i(p) - k_i + \int_{\underline{p}}^{\bar{p}} EG^*\left(\min\{z, p\}, C \setminus \{i\}\right) dF_i(p), \quad (6)$$

where

$$EG^*(z, C) = \max_{j \in C}\left\{EG(j; z, C), 0\right\}.$$

To determine the rational search decision is to compare the net expected gain of all the options in the choice set. If further search yields no positive net expected gain, stopping is the rational decision and $EG^* = 0$. Otherwise, the rational search decision is to continue to inspect the position that generates the highest net expected gain. In the case of a tie, randomly inspecting any of them is rational. We next use an example to illustrate consumers' rational search strategy.

> Example 2: Consider two firms. Firm 1 sets its price equal to 1 or 0 with equal probability. Firm 2 prices uniformly over [0, 1/2]. Consumers' inspection costs are the same for both firms, and let $k_1 = k_2 = 1/8$. The rational search strategy in this case is to inspect the first firm at first: if the quoted price is zero, stop searching; otherwise, continue to inspect the second firm.

In this example, the expected price of the first firm (1/2) is *higher* than the expected price of the second firm (1/4). This example shows that it may not be rational to start searching from the position with a lower expected price even if the inspection costs are the same. In fact, consumers' rational search strategies generally depend on the full distributions of equilibrium prices, which in turn are determined by consumers' rational search strategies. Such interdependence makes the analysis complex, especially when we consider asymmetric oligopolistic competition and heterogeneous consumer

search costs. For this reason, equilibrium analysis of oligopolistic pricing with rational search generally results in no closed-form solution. In this section, we seek to derive explicit equilibrium under certain conditions.

We consider the simplest oligopolistic case of three firms. The three firms are located in three different positions. Again, we refer to the firm located in the ith position as the ith firm or firm i, $i \in \{1, 2, 3\}$. Consumers are different in their search costs. To be consistent with the previous settings, we assume that among all consumers with total mass 1, α_1 of them have the highest search costs and are referred to as type 1 consumers; $\alpha_2(1 - \alpha_1)$ of them have lower search costs and are referred to as type 2 consumers; the rest $(1 - \alpha_1)(1 - \alpha_2)$ are shoppers. For the sake of simplicity, we let $\alpha_1 = \alpha_2 = \alpha$ $(0 < \alpha < 1)$. Assume type 1 consumers incur a cost k_i to inspect the ith position, while type 2 consumers incur a cost k_i', $i \in \{1, 2, 3\}$, $0 \le k_i' \le k_i \le 1$. All other settings follow the previous model setup.

We first study the case in which consumers' inspection costs are position-invariant; that is, $k_1 = k_2 = k_3$ and $k_1' = k_2' = k_3'$. In this case, there is essentially no difference among positions, and firms thus are symmetric. We derive symmetric equilibrium under certain conditions and uncover an interesting equilibrium pricing pattern that involves segmentation of price supports. We also show that the local-competition pattern from the previous analysis does not hold in the case of position-invariant inspection costs. We then allow inspection costs to vary across different positions and give a necessary condition and a sufficient condition for the local-competition pattern to arise in equilibrium.

Position-Invariant Costs

In this subsection, we consider the case that the inspection costs are the same for all positions. We let $k_1 = k_2 = k_3 \equiv k$ and $k_1' = k_2' = k_3' \equiv k'$, and we assume $0 < k' < k < 1$. Because we now do not impose any ordering constraint, when there is no difference in the inspection cost for different positions, all positions, and thus all firms, are essentially the same. Naturally, this case reduces to the symmetric random search setting. When nonshoppers have the same search cost, the symmetric equilibrium price distribution can be explicitly derived and analyzed [18]. When nonshoppers have heterogeneous search costs, like in our setting, the equilibrium analysis is more complicated because nonshoppers may adopt different search strategies, which in turn complicates firms' pricing decisions. Stahl [19] shows that with continuous cost distribution, a symmetric mixed-strategy equilibrium always exists, although the actual distribution patterns depend on the cost distribution and generally have no closed-form solutions. To better contrast with the local-competition pattern from the previous analysis, we next explicitly derive the symmetric equilibrium under certain conditions.

In the symmetric case, because firms adopt the same pricing strategies in a symmetric equilibrium, the search order becomes trivial: in each step, the rational strategy is either to stop or to randomly pick one of the uninspected firms. In other words, the search strategies simply reduce to the stopping rules. In addition, to determine the stopping rules, the expected gain in Equation (6) is easier to calculate. As we can show, when

firms adopt the same pricing strategy, the optimal stopping decision based on the expected gain from all the future searches is exactly the same as the decision based simply on the expected gain from the next one search. Therefore, in the symmetric equilibrium where each firm prices according to $F(\cdot)$, consumers' search strategies (with search cost k) can be characterized by an optimal stopping price r^*, which is determined by

$$\int_{\underline{p}}^{r^*} \left(r^* - p\right) dF\left(p\right) - k = 0. \tag{7}$$

When the current lowest price exceeds r^*, it is worthwhile to conduct an additional search; otherwise, it is optimal to stop. Thus, in the following symmetric equilibria, we consider consumers' rational search strategies as follows: shoppers always inspect all positions before making a purchase, and nonshoppers randomly inspect one firm with equal probability. If the price does not exceed their optimal stopping price (denote as r_1^* for type 1 consumers and r_2^* for type 2 consumers), they stop searching and purchase the product. Otherwise, they continue to randomly inspect one of the firms left with equal probability, until the price is no greater than their optimal stopping price or there is no firm left to inspect. They then buy from the one with the lowest price. Therefore, a complete description of the symmetric equilibrium only needs to specify firms' common pricing strategy (represented by the cumulative distribution function F) and nonshoppers' optimal stopping prices (r_1^* and r_2^*).

Proposition 2 (Symmetric Pricing with Continuous Support): There exists a mixed-strategy equilibrium in which all firms price according to the distribution

$$F\left(p\right) = 1 - \sqrt{\frac{\left[1 - \left(1 - \alpha\right)^2\right]\left(\bar{p}_1 - p\right)}{3\left(1 - \alpha\right)^2 p}} \quad p \in \left[\bar{p}_2, \bar{p}_1\right], \tag{8}$$

where $\bar{p}_1 = r_2^*$ and

$$\bar{p}_2 = \frac{1 - \left(1 - \alpha\right)^2}{1 + 2\left(1 - \alpha\right)^2} r_2^*,$$

and the optimal stopping prices $r_1^* = r_2^* + k - k'$ and r_2^* are determined by

$$\int_{\bar{p}_2}^{\bar{p}_1} F\left(p\right) dp = k',$$

if the inspection costs k and k' are not too different such that $0 < r_2^* < r_1^* < 1$ and $r_2^* > (1/(2 - \alpha)) r_1^*$.

The next example illustrates the equilibrium described in Proposition 2.

Example 3: When $\alpha = 0.3$, $k = 0.3$, and $k' = 0.2$, we can calculate nonshoppers' optimal stopping prices $r_1^* = 0.62$ and $r_2^* = 0.52$ ($r_2^* > (1/(2 - \alpha)) r_1^*$) and firms'
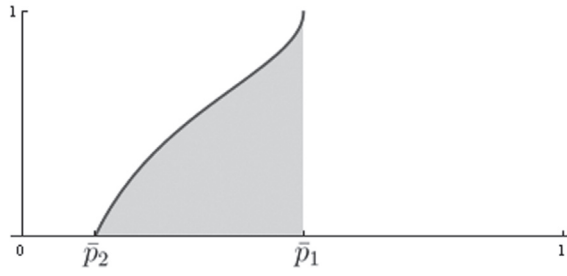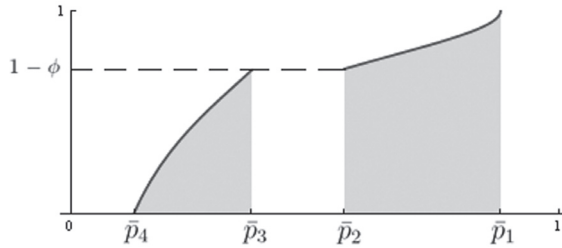
(a) $F(p)$ in Example 3

(b) $F(p)$ in Example 4

Figure 3. Examples of Price Distributions in the Symmetric Equilibrium

price support bounds $\bar{p}_1 = 0.52$ and $\bar{p}_2 = 0.13$. The price distribution is depicted in Figure 3a.

When the search cost of the type 1 consumers is not too high relative to that of the type 2 consumers, firms set the upper bound of the price support equal to the optimal stopping price of the type 2 consumer so that all nonshoppers stop searching after sampling once. In this case, the firms forgo the option of charging a higher price to take advantage of the high-cost type 1 consumers because the benefit from exploiting the high-cost consumers cannot counterbalance the loss of business from the low-cost consumers.

Proposition 3 (Symmetric Pricing with Segmented Support): There exists a mixed-strategy equilibrium in which all firms price according to the distribution

$$F(p) = \begin{cases} 1 - \sqrt{\dfrac{\alpha\bar{p}_1 - \alpha p - \alpha(1-\alpha)\left(1+\phi+\phi^2\right)p}{3(1-\alpha)^2 p}} & p \in \left[\bar{p}_4, \bar{p}_3\right] \\ 1 - \sqrt{\dfrac{\alpha\left(\bar{p}_1 - p\right)}{3(1-\alpha)p}} & p \in \left[\bar{p}_2, \bar{p}_1\right], \end{cases} \tag{9}$$

where

$$\bar{p}_1 = r_1^*$$

$$\bar{p}_2 = \frac{\alpha}{\alpha + 3(1-\alpha)\phi^2} r_1^*$$

$$\bar{p}_3 = r_2^*$$

$$\bar{p}_4 = \frac{\alpha}{\alpha + 3(1-\alpha)^2 + \alpha(1-\alpha)(1+\phi+\phi^2)} r_1^*$$

and

$$\phi = \frac{-\alpha}{2(3-2\alpha)} + \frac{\sqrt{\alpha(-9\alpha^2 + 29\alpha - 24) + 4\alpha(3-2\alpha)r_1^*/r_2^*}}{2\sqrt{1-\alpha}(3-2\alpha)},$$

and the optimal stopping prices r_1^* and r_2^* are determined by

$$\begin{cases} \int_{\bar{p}_4}^{\bar{p}_3} F(p)\,dp = k' \\ \int_{\bar{p}_2}^{\bar{p}_1} F(p)\,dp + (\bar{p}_2 - \bar{p}_3)(1-\phi) = k - k', \end{cases} \tag{10}$$

if the inspection costs k and k′ are different enough such that $0 < r_2^* < r_1^* < 1$ and

$$\frac{\alpha}{3-2\alpha} r_1^* < r_2^* < \frac{1}{2-\alpha} r_1^*.$$

The next example illustrates the equilibrium described in Proposition 3.

Example 4: When $\alpha = 0.3$, $k = 0.5$, and $k' = 0.1$, we can calculate nonshoppers' optimal stopping prices $r_1^* = 0.88$ and $r_2^* = 0.37$ ($[\alpha/(3-2\alpha)]r_1^* < r_2^* < [1/(2-\alpha)]r_1^*$) and firms' price support $\bar{p}_1 = 0.88$, $\bar{p}_2 = 0.56$, $\bar{p}_3 = 0.37$, and $\bar{p}_4 = 0.13$. Also, $\phi = 0.29$. The price distribution is depicted in Figure 3b.

Intuitively, when the search cost of the type 1 consumers is significantly higher than that of type 2 consumers, the optimal stopping price of the type 1 consumers is thus considerably higher than that of the type 2 consumers. As a result, charging a higher price to exploit the high-cost consumers could be as profitable as charging a lower price to compete for more market share, which explains the rightward expansion of the price support compared to the previous result. Particularly, it is worth noting that *segmentation* of price supports arises in equilibrium in this case. The gap between $\bar{p}_3$ and $\bar{p}_2$ results from the drop of expected demand for prices right above $\bar{p}_3$. Because $\bar{p}_3$ equals the type 2 consumers' optimal stopping price r_2^*, when pricing below $\bar{p}_3$, a firm can stop all type 2 consumers who inspect its position from further searching. However, once its price exceeds $\bar{p}_3$, the type 2 consumers will continue to inspect other positions and are very likely to purchase from elsewhere (unless all the other positions charge even higher prices). Therefore, the expected demand drops substantially at $\bar{p}_3$. As a

result, the expected profit jumps downward at $\bar{p}_3$ and does not rise back until $p \geq \bar{p}_2$. For this reason, charging any price between $\bar{p}_3$ and $\bar{p}_2$ is suboptimal.

One question of our particular interest is: could the local-competition price pattern arise in equilibrium in the case of position-invariant costs? The answer is negative, as we show in Proposition 4. In fact, we can conclude that the typical symmetric random search market cannot induce the local-competition equilibrium pricing pattern.

Position-Dependent Costs

We now allow consumers' inspection costs to be different for different positions. Position-dependent inspection costs reflect the inherent difference across different positions, which lies in the difference in terms of visibility and accessibility. Just like bending over to check the bottom-level shelf space in the supermarket could be costly for seniors, scrolling down and looking for an unhighlighted link on a Web page or switching multiple Web pages could be troublesome for non-tech-savvy users or in the case of unsatisfactory network connection.

We first give a necessary condition for the local-competition pattern to arise in equilibrium:

> *Proposition 4 (Local-Competition Pattern Under Endogenous Search: A Necessary Condition): A similar pricing pattern as in Equation (1) may arise in a rational-expectations equilibrium only if at least one group of consumers have a higher inspection cost for position 3 than for position 1 (i.e., $k_3 > k_1$ or $k_3' > k_1'$).*

Proposition 4 shows that for the pattern to appear in REE, at least some consumers' inspection cost for the third position should be strictly higher than that for the first position. Otherwise, because the price at the first position is always higher than that at the third position, as in Equation (1), inspecting the first position would be dominated by inspecting the third one for all consumers. In this case, the rational search decision would be to inspect the second and the third positions only. If this is the case, then both the first and the second firms will deviate from the presumed pricing strategies. As a result, the pattern in Equation (1) cannot hold as an equilibrium.

Proposition 4 thus excludes the possibility that the local-competition pattern is an equilibrium when there is no inherent difference among different positions. It reveals the fact that the special pattern of equilibrium price dispersion is an outcome of position-dependent inspection costs, which induce certain search ordering of consumers as rational equilibrium behaviors.

We next explicitly derive an equilibrium with the same pricing pattern as Equation (1) under certain conditions:

> *Proposition 5 (Local-Competition Pattern Under Endogenous Search: A Sufficient Condition): When nonshoppers' inspection costs for different positions increase with position ranks and the differences are large enough, precisely when*

$$k_1 < \frac{\alpha(1-\alpha) - \ln(1+\alpha(1-\alpha))}{1+\alpha(1-\alpha)} \bar{p}_1$$

$$k_3 - k_2 > \bar{p}_3 \frac{\alpha}{1-\alpha} \ln \alpha + \frac{\bar{p}_1}{1-\alpha} \ln \left(1 + \alpha \left(1 - \alpha\right)\right)$$

$$k_1' = 0$$

$$k_2' < \left(\gamma^* - \bar{p}_3\right) - \bar{p}_3 \ln \frac{\gamma^*}{\bar{p}_3}$$

with

$$\gamma^* = \frac{1 + 2\alpha\left(1-\alpha\right) + \sqrt{4\alpha^4 - 12\alpha^3 + 12\alpha^2 - 4\alpha + 1}}{2\left(2-\alpha\right)\left[1 + \alpha\left(1-\alpha\right)\right]} \bar{p}_1$$

and

$$k_3' > \left[1 + \frac{\alpha \ln \alpha}{\left(1-\alpha\right)\left[1 + \alpha\left(1-\alpha\right)\right]}\right] \bar{p}_1,$$

there exists a rational-expectations equilibrium in which firms price according to

$$F_1(p) = \begin{cases} 1 - \dfrac{\bar{p}_2}{p} & p \in \left[\bar{p}_2, \bar{p}_1\right) \\ 1 & p = \bar{p}_1 \end{cases}$$

$$F_2(p) = \begin{cases} 1 - \dfrac{\bar{p}_3}{p} & p \in \left[\bar{p}_3, \bar{p}_2\right) \\ 1 - \dfrac{\left(\bar{p}_1 - p\right)}{\left(1-\alpha\right)p} & p \in \left[\bar{p}_2, \bar{p}_1\right] \end{cases} \qquad (11)$$

$$F_3(p) = 1 - \frac{\alpha\left(\bar{p}_2 - p\right)}{\left(1-\alpha\right)p} \quad p \in \left[\bar{p}_3, \bar{p}_2\right],$$

where

$$\bar{p}_1 = \min \left\{ k_2 \left[1 + \frac{\alpha \ln \alpha}{1 + \alpha\left(1-\alpha\right)} - \frac{\ln\left(1 + \alpha\left(1-\alpha\right)\right)}{1-\alpha}\right]^{-1}, 1 \right\}$$

$$\bar{p}_2 = \frac{1}{1 + \alpha\left(1-\alpha\right)} \bar{p}_1,$$

and

$$\bar{p}_3 = \alpha \bar{p}_2;$$

consumers adopt the search strategies specified in Table 1.[3]

Table 1. Consumers' Search Decisions $d(z, C)$

Choice set C	Type 1 consumers	Type 2 consumers
{1,2,3}	If $z = 1$, inspect first position	If $z = 1$, inspect first position
{2,3}	If $(1 \geq) z > \bar{p}_1$, inspect second position; If $z \leq \bar{p}_1$, stop	If $z > r_2$, inspect second position; If $z \leq r_2$, stop
{1,3}	If $z > r_1$, inspect first position; If $z \leq r_1$, stop	If $z > \bar{p}_2$, inspect first position
{1,2}	If $z \leq \bar{p}_2$, stop	If $z > \bar{p}_2$, inspect first position
{1}	If $z \leq r_1$, stop	If $z > \bar{p}_2$, inspect first position
{2}	If $z \leq \bar{p}_1$, stop	If $z > r_2$, inspect second position; If $z \leq r_2$, stop
{3}	If $z \leq \bar{p}_1$, stop	If $z \leq \bar{p}_1$, stop

r_1 is defined by

$$\int_{\bar{p}_2}^{r_1} (r_1 - p)\, dF_1(p) = k_1. \text{ (Notice that } \bar{p}_2 < r_1 < \bar{p}_1.)$$

r_2 is defined by

$$\int_{\bar{p}_3}^{r_2} (r_2 - p)\, dF_2(p) = k_2'. \text{ (Notice that } \bar{p}_3 < r_2 < \bar{p}_2.)$$

The conditions needed are that the inspection costs are higher for the inferior positions than those of the superior ones and that they differ in the same order for all nonshoppers (i.e., $k_1 < k_2 < k_3$ and $k_1' < k_2' < k_3'$). In addition, the differences between the inspection costs for two different positions are heterogeneous across consumers. In other words, while some consumers (i.e., shoppers) are insensitive to location difference, others (i.e., nonshoppers) are quite sensitive, and these nonshoppers have different levels of tolerance toward locational inconvenience. For example, some consumers might not mind scrolling down and looking for an unhighlighted link on the same Web page but would not bother to switch to another Web page to continue their search, or it might be fine for some to bend over and check the bottom shelf in the supermarket but it would be too troublesome to get a ladder and check the top shelf. As an equilibrium outcome of such differences, a commonly observed search order arises in equilibrium, and meanwhile different consumers stop at different stages of the search process.

Anticipating consumers' rational search strategies, firms also optimize their pricing strategies by selectively retaining some consumers from further search, competing for some who sample multiple positions and forgoing the others. As an equilibrium outcome, type 1 consumers choose to start their search from the first position because it yields the highest expected gain according to Equation (6). After learning the price from the first position, they decide to stop searching because further inspection of either the second or the third position yields a negative expected gain. Similarly, type 2 consumers stop searching after inspecting the first two positions because of the higher

search cost for the third position. Shoppers inspect all three positions before making a purchase, and the order they pursue actually does not matter.

Table 1 lists nonshoppers' search decisions $d(z, C)$ when facing values of $\{z, C\}$ that are relevant to the equilibrium analysis. From the table, we can identify nonshoppers' equilibrium search strategies. For instance, the lowest price is $z = 1$ before consumers start the first search, and the decision facing consumers is $d(1, \{1, 2, 3\})$. For type 1 consumers, according to Table 1, $d(1, \{1, 2, 3\})$ is to inspect the first position. After that, they learn a price p from the $F_1(p)$ so that $p \in [\bar{p}_2, \bar{p}_1]$. Then the decision $d(p, \{2, 3\})$ for the type 1 consumers is to stop, which completes the type 1 consumers' search strategies in equilibrium. We also specify consumers' off-equilibrium strategies. For instance, the type 1 consumers will not actually face the decision $d(z, \{1, 3\})$. Nevertheless, this decision and the associated payoff should be taken into account when they calculate the expected gain of inspecting the second position in the first place. Also, the type 2 consumers face the decision $d(z, \{2, 3\})$ after inspecting the first position. Despite the fact that the price quoted from the first position always falls in $[\bar{p}_2, \bar{p}_1]$ in equilibrium, we also need to specify the off-equilibrium search decision when $z < \bar{p}_2$ because such decision affects the first firm's profitability of possible deviation. As we can verify, all the decisions listed in Table 1, both in-equilibrium and off-equilibrium, are rational.

Conclusion

IN ADDITION TO THE THEORETICAL CONTRIBUTION to the literature on search and pricing, this paper has managerial implications for sellers, consumers, and information systems designers. For sellers seeking the optimal pricing strategy, we emphasize the importance of recognizing the features of consumers' online search behavior and adjusting the price accordingly. Because the online environment makes it relatively easy to sample around, a competitive price could quickly be noticed and result in a surge of sales. Therefore, it would be beneficial to provide occasional promotions or limited-time deals from time to time, which could not only boost short-term sales but also attract more visits in the long run. Nevertheless, the optimal pricing strategies would avoid being too aggressive or too conservative in price competition. The local-competition pattern suggests competing only against commensurable opponents with similar visibility. In most cases, it would be suboptimal to compete against sellers that are much stronger or weaker. Moreover, to form their optimal pricing strategies, sellers only need "local" information on consumers' search behavior at the positions with similar visibility and on the pricing strategies of the firms at these neighboring positions.

For consumers looking for the best deals, it might sound discouraging that the exact price at a particular position is usually difficult to predict. It could be the case that the firm at a prominent position offers a good deal, or a firm at an inferior position might not charge as low a price as expected. Stopping the search process early is thus rational for consumers who have high search costs. Nevertheless, since the price expectation decreases as the location prominence drops, it is generally rewarding to keep searching, especially for those who are not sensitive to locational inconvenience.

For designers of online information portals, this study highlights the critical role of the inherent difference among business positions in affecting consumers' inspection costs, which then determine the search behavior and, in turn, sellers' profits. Appropriately differentiating the visibility of links (e.g., by special decoration, large display areas, or color highlighting) is essential to make the superior positions more appealing to sellers. In addition, notice that the advantage of prominent positions would diminish once consumers get fully accustomed to the Web page design and linking structure. In this sense, occasionally updating the structural design would also be necessary.

This work can be extended in several directions. The major limitation of the analysis in the fourth section is that it is somewhat incomplete, as we only study the equilibrium with full rationality in the three-firm case under certain conditions. A future extension would be to explore the general case of asymmetric oligopolistic pricing with optimal consumer search. Another interesting direction for future work would be to consider a dynamic setting in which consumers arrive at different times, and firms compete against each other intertemporally. In addition, the local-competition pattern predicted from the current study could be an interesting topic for future empirical investigation of online price dispersion.

NOTES

1. Peterson and Merino [15] argue that in reality consumers search in a way different from the rational assumptions in economic theory, and this situation continues in the online world.

2. When the condition does not hold, which implies that a lower position retains a larger portion of the visitors than an upper one, the firm at the upper position may deviate to a lower price range.

3. Alternatively, we can describe consumers' search strategies based on *Pandora's rule,* which relies on a major result from Weitzman [22]. Here, we opt to formulate and solve the whole problem within a self-contained framework.

4. The only exception is that there will be a downward jump in expected profit at the upper bound $\bar{p}_1$ for the firm in position 2, which is incurred by the mass point in $F_1(\cdot)$ at $\bar{p}_1$. However, a jump down at one single point with zero probability measure does not affect the actual profit.

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Appendix A: Summary of Notation

Notation	Definition	Comments
α_i	Among the consumers inspecting position i, the portion who stop searching thereafter	$\alpha_0 = 0$, $0 < \alpha_i < 1$ ($i = 1,\dots,n-1$)
β_i	The total number of consumers who inspect position i	$\beta_1 = 1$, $\beta_i = \Pi_{j=1}^{i-1}(1 - \alpha_j)$ ($i = 2, \dots, n$)
$F_i(p)$	The probability that firm i charges a price less than or equal to p	The cumulative distribution function of firm i's pricing
$\bar{p}_1, \dots, \bar{p}_n$	The bounds of firms' price supports	$\bar{p}_n < \dots < \bar{p}_1$
π_i	Firm i's expected profit	
z	The lowest price from all the inspected positions	When no position has been inspected yet, $z = 1$
C	The choice set containing all the uninspected positions	
$d(z, C)$	The search decision when the lowest inspected price is z and the choice set is C	Each decision is to stop, to inspect a particular position, or to randomly inspect several positions with a probability distribution
$EG(i; z, C)$	The expected gain from inspecting position i when the lowest inspected price is z and the choice set is C	
k_1, k_2, k_3	The inspection costs of the type 1 (or the high-cost) consumers for positions 1, 2, and 3	When $k_1 = k_2 = k_3$, we suppress the subscript and denote them as k
k_1', k_2', k_3'	The inspection costs of the type 2 (or the low-cost) consumers for positions 1, 2, and 3	When $k_1' = k_2' = k_3'$, we suppress the subscript and denote them as k'
r^*	Consumers' optimal stopping price	When the quoted price exceeds r^*, consumers continue searching; otherwise, consumers stop searching

Appendix B

Proof of Proposition 1

Denote $F_i(p)$ as $F_i^-(p)$ for $p \in [\bar{p}_{i+1}, \bar{p}_i)$ and as $F_i^+(p)$ for $p \in [\bar{p}_i, \bar{p}_{i-1}]$.

(1) First, $F_i(\cdot)$, $i = 1, ..., n$, is a well-defined cumulative distribution function. Notice that all supports are well defined because $0 < k_i < 1$, $i = 1, ..., n - 1$, and thus $\{\bar{p}_i\}_{i=1}^n$ is positive and monotonically decreasing.

Each $F_i^-(\cdot)$ or $F_i^+(\cdot)$ is strictly increasing within its support, and $F_i^-(\bar{p}_i) = F_i^+(\bar{p}_i)$ because

$$1 - F_i^- \left(\bar{p}_i \right) = \frac{\bar{p}_{i+1}}{\bar{p}_i} = k_i = \frac{\alpha_{i-1}}{\alpha_i \left(1 - \alpha_{i-1} \right)} \left(\frac{1}{k_{i-1}} - 1 \right)$$

$$= \frac{\alpha_{i-1}}{\alpha_i \left(1 - \alpha_{i-1} \right)} \frac{\left(\bar{p}_{i-1} - \bar{p}_i \right)}{\bar{p}_i} = 1 - F_i^+ \left(\bar{p}_i \right).$$

Therefore, each $F_i(\cdot)$ is strictly increasing in its entire support. Because $F_i(\bar{p}_{i+1}) = 0$ and $F_i(\bar{p}_{i-1}) = 1$, $F_i(\cdot)$ is well defined.

(2) Next, we show that each position i yields a constant expected profit π_i within the entire support $[\bar{p}_{i+1}, \bar{p}_{i-1}]$.

If the firm in position i $(i = 2, ..., n - 1)$ prices at $p \in [\bar{p}_i, \bar{p}_{i-1}]$, it only competes with the firm in position $i - 1$ for the demand $\alpha_i \beta_i$. (Recall that $\beta_i = \Pi_{j=1}^{i-1}(1 - \alpha_j)$.) Thus, the expected profit is $\pi_i^+(p) = [1 - F_{i-1}^-(p)] \cdot p \cdot \alpha_i \beta_i \equiv \alpha_i \beta_i \bar{p}_i$. If it prices in the lower interval $[\bar{p}_{i+1}, \bar{p}_i]$, it will win over the firm above and capture the demand $\alpha_i \beta_i$, and meanwhile compete with the firm below for the demand $\alpha_{i+1} \beta_{i+1}$, which leads to the expected profit:

$$\pi_i^- \left(p \right) = p \cdot \alpha_i \beta_i + \left[1 - F_{i+1}^+ \left(p \right) \right] \cdot p \cdot \alpha_{i+1} \beta_{i+1}$$

$$\equiv \alpha_i \beta_i \bar{p}_i.$$

Therefore, $\pi_i^+(p) = \pi_i^-(p)$; that is, the expected profit remains constant over the entire support.

The firm at position n competes only with the firm above it. A calculation similar to that for the above $\pi_i^+(p)$ shows a constant expected profit $\beta_n \bar{p}_n$. Similarly, we can verify the constant expected profit for the firm at position 1.[4]

(3) Now we verify that no unilateral profitable deviation exists by pricing outside the given support.

Suppose the firm in position i (≥ 3) prices at $p > \bar{p}_{i-1}$. If $p < \bar{p}_{i-2}$, it faces a competition with the two firms above, and the expected profit is

$$\pi_i \left(p \right) = \left[1 - F_{i-1}^+ \left(p \right) \right] \left[1 - F_{i-2}^- \left(p \right) \right] \cdot p \cdot \alpha_i \beta_i = \alpha_i \beta_i \frac{\alpha_{i-2} \left(\bar{p}_{i-2} - p \right)}{\alpha_{i-1} \left(1 - \alpha_{i-2} \right)} \frac{\bar{p}_{i-1}}{p},$$

which is strictly decreasing in p; that is, $\pi_i(p) < \pi_i(\bar{p}_{i-1})$, $\forall p \in (\bar{p}_{i-1}, \bar{p}_{i-2})$. If $p \geq \bar{p}_{i-2}$, then the firm would have zero demand (and thus zero profit) because the firm right above

it charges a lower price for sure. The firms at positions 1 and 2 will clearly not charge a price higher than $\bar{p}_1$, since $\bar{p}_1 = w$, consumers' willingness to pay. Thus, there is no unilateral profitable deviation by pricing higher beyond the given support.

Suppose the firm in position i ($\leq n - 2$) prices at $p < \bar{p}_{i+1}$. If $p \in [\bar{p}_{i'+1}, \bar{p}_{i'}]$ ($i' \geq i + 1$), it could capture all consumers who stop sampling at any position from i to $i' - 1$. We denote the amount of those consumers as $D_{ii'}$. In addition, it competes with the firms in positions i' and $i' + 1$ for consumers who stop sampling at positions i' and $i' + 1$. Thus, the expected profit can be written as

$$\pi_i(p) = D_{ii'} p + \left[1 - F_{i'}^-(p)\right] p \alpha_{i'} \beta_{i'} + \left[1 - F_{i'}^-(p)\right]\left[1 - F_{i'+1}^+(p)\right] p \alpha_{i'+1} \beta_{i'+1}$$

$$= D_{ii'} p + \left[1 - F_{i'}^-(p)\right] \cdot \alpha_{i'} \beta_{i'} \bar{p}_{i'}$$

$$= D_{ii'} p + \alpha_{i'} \beta_{i'} \bar{p}_{i'} \frac{\bar{p}_{i'+1}}{p},$$

where the second and third steps are derived by substituting in $F_{i'+1}^+(p)$ and $F_{i'}^-(p)$ from Equation (1), respectively. Note that $D_{ii'} \geq \alpha_{i'-1} \beta_{i'-1} \geq \alpha_{i'} \beta_{i'}$, where the last inequality is due to the assumption $\alpha_i \geq \alpha_{i+1}(1 - \alpha_i)$. Therefore, $\pi_i(\bar{p}_{i'}) - \pi_i(\bar{p}_{i'+1}) = (\bar{p}_{i'} - \bar{p}_{i'+1})(D_{ii'} - \alpha_{i'} \beta_{i'}) \geq 0$. Also noticing that $\pi_i(p)$ is convex, we can conclude that $\pi_i(p) < \pi_i(\bar{p}_{i'})$, $\forall p \in (\bar{p}_{i'+1}, \bar{p}_{i'})$. Iteratively, we get $\pi_i(p) \leq \pi_i(\bar{p}_{i+1})$, $\forall p < \bar{p}_{i+1}$. For firms at position $n - 1$ and n, clearly, it is unprofitable to price below $\bar{p}_n$, since otherwise the profit margin would decrease without an increase in demand. Hence, there is no unilateral profitable deviation by pricing below the given support.

Combining items (2) and (3) above , we conclude that given other firms' strategies, each firm is indifferent in pricing over the given support and has no profitable deviation. Therefore, the price strategy described in the proposition is a mixed-strategy equilibrium.

Proof of Corollary 2

For $i = 2, ..., n - 1$, denote $E_i^-(p_i)$ and $E_i^+(p_i)$ as the price expectations from position i, *conditional on* $p_i \in [\bar{p}_{i+1}, \bar{p}_i]$ and *conditional on* $p_i \in [\bar{p}_i, \bar{p}_{i-1}]$, respectively. Also denote $f_i^-(\cdot)$ and $f_i^+(\cdot)$ as the corresponding conditional probability density functions. According to Equation (1), both $f_i^-(p)$ and $f_{i+1}^+(p)$ take the form of K/p^2 on the same support $[\bar{p}_{i+1}, \bar{p}_i]$ and thus must be identical. (It can also be calculated that K is $\bar{p}_i \bar{p}_{i+1}/ (\bar{p}_i - \bar{p}_{i+1})$ in the former and is

$$\frac{\alpha_i}{\alpha_{i+1}(1 - \alpha_i)} \frac{\bar{p}_i \bar{p}_{i+1}}{\bar{p}_{i+2}}$$

in the latter. They are the same due to Equations (2) and (3)). Therefore, $E_i^-(p_i) = E_{i+1}^+(p_{i+1})$. Notice that the expected price from the ith position $E(p_i)$ is the weighted average of $E_i^-(p_i)$ and $E_i^+(p_i)$, and that from the $i + 1$th position, $E(p_{i+1})$ is the weighted average of $E_{i+1}^-(p_{i+1})$ and $E_{i+1}^+(p_{i+1})$. Because $E_i^-(p_i) = E_{i+1}^+(p_{i+1})$ and $E_i^+(p_i) > E_{i+1}^-(p_{i+1})$, $E(p_i) > E(p_{i+1})$. Similar arguments apply to position 1 and position n.

Proof of Proposition 2

First, we show that given consumers' search strategies (i.e., nonshoppers' optimal stopping prices r_1^* and r_2^*), and other firms' pricing strategies as in Equation (8), no firm has profitable deviation in pricing. When pricing any $p \in [\bar{p}_2, \bar{p}_1]$, a firm achieves a constant expected profit

$$\pi(p) = \frac{1}{3}\left[\alpha + \alpha(1-\alpha)\right]p + (1-\alpha)^2 p\left[1 - F(p)\right]^2 \equiv \frac{1}{3}\left[\alpha + \alpha(1-\alpha)\right]\bar{p}_1$$

by substituting in $F(p)$. If a firm deviates to charge a price equal to r_1^*, it forgoes the type 2 consumers while still retaining the type 1 consumers. Its expected profit would be

$$\pi'\left(r_1^*\right) = \frac{1}{3}\alpha r_1^*.$$

Notice that

$$r_2^* > \frac{1}{2-\alpha}r_1^*,$$

which implies

$$\pi'\left(r_1^*\right) < \frac{1}{3}\left[\alpha + \alpha(1-\alpha)\right]\bar{p}_1.$$

Because any price $p \in (r_2^*, r_1^*)$ yields an even lower expected profit and any price $p > r_1^*$ results in zero profit, there is no profitable deviation from pricing above $\bar{p}_1$. Also, there is no profitable deviation from underpricing, because any $p < \bar{p}_2$ yields a lower profit level than

$$\frac{1}{3}\left[\alpha + \alpha(1-\alpha)\right]\bar{p}_1.$$

Second, we show that given firms' pricing strategies in Equation (8), r_1^* and r_2^* are rational. By Equation (7), consumers' rationality requires

$$\begin{cases} \int_{\bar{p}_2}^{r_2^*}\left(r_2^* - p\right)dF(p) - k' = 0 \\ \int_{\bar{p}_2}^{r_1^*}\left(r_1^* - p\right)dF(p) - k = 0, \end{cases}$$

which is ensured by

$$\int_{\bar{p}_2}^{\bar{p}_1} F(p)\,dp = k'$$

and $r_1^* = r_2^* + k - k'$. Notice that because $r_2^* < r_1^* < 1$, it is rational for both types of nonshoppers to enter the market.

Altogether, the strategy profile specified in Proposition 2 is an equilibrium.

Proof of Proposition 3

First, $F(p)$ in Equation (9) is a well-defined cumulative distribution function. We can verify that $F(\bar{p}_4) = 0$, $F(\bar{p}_1) = 1$, and F is increasing in the two segments of the support. Also, $F(\bar{p}_3) = F(\bar{p}_2) = 1 - \phi$. To see this, first notice that ϕ is the solution to the quadratic equation

$$\bar{p}_3\left[\alpha + \alpha(1-\alpha)\left(1 + \phi + \phi^2\right) + 3(1-\alpha)^2 \phi^2\right] = \alpha\bar{p}_1. \tag{B1}$$

Let the left-hand side of Equation (B1) be $f(\phi)$. The condition

$$\frac{\alpha}{3 - 2\alpha} r_1^* < r_2^* < \frac{1}{2-\alpha} r_1^*$$

ensures that $f(0) < \alpha\bar{p}_1$ and $f(1) > \alpha\bar{p}_1$. Also, the quadratic coefficient of $f(\phi)$ is positive. Hence, $\phi \in (0, 1)$ is well defined. Therefore, substituting Equation (B1) and the definition of $\bar{p}_2$ into the first and second equations of Equation (9), respectively, we have $F(\bar{p}_3) = F(\bar{p}_2) = 1 - \phi$. The price support is also well defined. $\bar{p}_2 > \bar{p}_3$ because

$$\bar{p}_3 = \frac{\alpha\bar{p}_1}{\alpha + \alpha(1-\alpha)\left(1 + x + x^2\right) + 3(1-\alpha)^2 x^2}$$

by Equation (B1) and its denominator is greater than that of $\bar{p}_2$ (i.e., $\alpha + \alpha(1-\alpha)[1 + x + x^2] + 3(1-\alpha)^2 x^2 - \alpha - 3(1-\alpha)x^2 = \alpha(1-\alpha)[(1-x^2) + x(1-x)] > 0$). Similarly, comparing the denominators of $\bar{p}_3$ and $\bar{p}_4$, we have $\bar{p}_3 > \bar{p}_4$.

Next, we show that given consumers' optimal stopping prices (i.e., r_1^* and r_2^*), and other firms' pricing strategies, no firm has profitable deviation in pricing. When pricing $p \in [\bar{p}_4, \bar{p}_3]$, a firm's expected profit function can be written as

$$\pi(p) = \frac{1}{3}\alpha p + \frac{1}{3}\alpha(1-\alpha)\left[1 + \left(1 - F\left(r_2^*\right)\right) + \left(1 - F\left(r_2^*\right)\right)^2\right]p + (1-\alpha)^2\left[1 - F(p)\right]^2 p,$$

where the first term on the right-hand side is the expected revenue from the type 1 consumers, the second term is that from the type 2 consumers, and the third term is that from shoppers. In particular, the expected revenue from the type 2 consumers consists of three parts: those who inspect this firm first and stop there $((1/3)\alpha(1 - \alpha))$, those who inspect another firm first but find its price exceeding r_2^* and continue to inspect this firm

$$\left(2 \times \frac{1}{3}\alpha(1-\alpha)\left(1 - F\left(r_2^*\right)\right) \times \frac{1}{2}\right),$$

and those who inspect the other two firms first but find both their prices exceeding r_2^* and continue to inspect this firm

$$\left(2 \times \frac{1}{3}\alpha(1-\alpha)\left(1 - F\left(r_2^*\right)\right)\frac{1}{2}\left(1 - F\left(r_2^*\right)\right)\right).$$

Substituting $F(p)$ from the first equation of Equation (9) (with $F(r_2^*) = 1 - \phi$) into the above profit function, we have $\pi(p) \equiv (1/3)\alpha\bar{p}_1$, $\forall p \in [\bar{p}_4, \bar{p}_3]$. Similarly, when pricing $p \in [\bar{p}_2, \bar{p}_1]$, a firm's expected profit function can be written as

$$\pi(p) = \frac{1}{3}\alpha p + (1-\alpha)[1 - F(p)]^2 p,$$

where the first term on the right-hand side is the expected revenue from the type 1 consumers and the second term is that from the type 2 consumers and the shoppers. Substituting in $F(p)$ from the second equation of Equation (9), we have $\pi(p) \equiv (1/3)\alpha\bar{p}_1$, $\forall p \in [\bar{p}_2, \bar{p}_1]$. Therefore, pricing within the support leads to a constant expected profit. In addition, any price $p \in (\bar{p}_3, \bar{p}_2)$ yields a lower profit than $\pi(\bar{p}_2)$. Any price $p > \bar{p}_1$ yields zero profit. Any price $p < \bar{p}_4$ yields a lower profit than $\pi(\bar{p}_4)$. Therefore, there is no profitable unilateral deviation for any firm.

Given firms' pricing strategies, r_1^* and r_2^* are rational because the rationality requirements

$$\begin{cases} \int_{\bar{p}_4}^{\bar{p}_3} (r_2^* - p)\,dF(p) = k' \\ \int_{\bar{p}_4}^{\bar{p}_3} (r_1^* - p)\,dF(p) + \int_{\bar{p}_2}^{\bar{p}_1} (r_1^* - p)\,dF(p) = k \end{cases}$$

are ensured by Equation (10). Notice that because $r_2^* < r_1^* < 1$, it is rational for both types to enter the market.

Altogether, the strategy profile specified in Proposition 3 is an equilibrium.

Proof of Proposition 4

To prove by contradiction, suppose that $k_1 \geq k_3$ and $k_1' \geq k_3'$ and that the price pattern in Equation (1) is an equilibrium. Then for both the type 1 and the type 2 consumers, inspecting the first position is dominated by inspecting the third one, because the third position offers a lower price for sure, while it does not incur a higher search cost. Thus, the rational search strategy should not involve inspecting the first position. In this case, the first firm has zero profit by charging $p \in [\bar{p}_2, \bar{p}_1]$ and will deviate to charge a lower price $p' \in [\bar{p}_3, \bar{p}_2)$, which gives positive expected profit. Also, given firm 3's price support, no matter what search order consumers choose between firm 2 and firm 3, it is suboptimal for firm 2 to price $p \in (\bar{p}_2, \bar{p}_1)$. In any case, the given pricing pattern cannot hold in equilibrium if $k_1 \geq k_3$ and $k_1' \geq k_3'$.

Proof of Proposition 5

First, we show that given consumers' search strategies and other firms' pricing strategies, no firm has profitable unilateral deviation. For firm 1, charging $\forall p \in [\bar{p}_2, \bar{p}_1]$ gives a constant expected profit

$$\pi_1(p) = \alpha p + \alpha(1-\alpha)p[1 - F_2(p)] \equiv \alpha\bar{p}_1$$

by substituting in firm 2's pricing strategies. Similarly, we can show that firm 2 has a constant expected profit by charging any price within $[\bar{p}_3, \bar{p}_1)$ because

$$\pi_2(p) = \alpha(1-\alpha)p\left[1 - F_1(p)\right] \equiv \alpha(1-\alpha)\bar{p}_2 \qquad\qquad p \in (\bar{p}_2, \bar{p}_1)$$

$$\pi_2(p) = \alpha(1-\alpha)p + (1-\alpha)^2 p\left[1 - F_3(p)\right] \equiv \alpha(1-\alpha)\bar{p}_2 \quad p \in [\bar{p}_3, \bar{p}_2].$$

Notice that $\pi_2(\bar{p}_1) < \alpha(1-\alpha)\bar{p}_2$ because of the mass point in F_1 at the upper bound. Nevertheless, because F_2 places no mass on $\bar{p}_1$, firm 2's expected profit is unaffected. Firm 3 also has constant profit by charging any price within its support:

$$\pi_3(p) = (1-\alpha)^2 p\left[1 - F_2(p)\right] \equiv (1-\alpha)^2 \bar{p}_3 \quad p \in [\bar{p}_3, \bar{p}_2].$$

If firm 3 deviates to charge $p \in (\bar{p}_2, \bar{p}_1)$, its profit would be

$$\pi_3'(p) = (1-\alpha)^2 p\left[1 - F_2(p)\right]\left[1 - F_1(p)\right] = (1-\alpha)\bar{p}_2 \frac{(\bar{p}_1 - p)}{p},$$

which is decreasing in p. Therefore, pricing above $\bar{p}_2$ is not profitable for firm 3. Also, firm 1 cannot achieve a higher profit by deviating to charge $p \in [\bar{p}_3, \bar{p}_2)$. To see this, first notice that after inspecting the first position, type 2 consumers' search strategy, $d(p, \{2,3\})$, is to inspect the second position if the price learned from firm 1 exceeds r_2, and to stop and buy from firm 1 otherwise. Here, r_2 $(\bar{p}_3 < r_2 < \bar{p}_2)$ is defined by

$$\int_{\bar{p}_3}^{r_2} (r_2 - p)dF_2(p) = k_2'.$$

When $p \in (r_2, \bar{p}_2)$, type 2 consumers continue to inspect the second position, and firm 1's profit would be

$$\pi_1'(p) = \alpha p + \alpha(1-\alpha)p\left[1 - F_2(p)\right] + (1-\alpha)^2 p\left[1 - F_2(p)\right]\left[1 - F_3(p)\right]$$

$$= \alpha p + \alpha(1-\alpha)\bar{p}_3 + (1-\alpha)^2 \bar{p}_3 \frac{\alpha(\bar{p}_2 - p)}{(1-\alpha)p},$$

which is convex in p. Because $\pi_1'(\bar{p}_3) = \bar{p}_3 < \alpha\bar{p}_1 = \pi_1'(\bar{p}_2)$, we can conclude that $\pi_1'(p) < \pi_1(\bar{p}_2)$ for $\forall p \in (r_2, \bar{p}_2)$. When $p \in [\bar{p}_3, r_2]$, type 2 consumers stop searching and buy from firm 1, and firm 1's profit function thus becomes

$$\pi_1''(p) = \left[\alpha + \alpha(1-\alpha)\right]p + (1-\alpha)^2 p\left[1 - F_2(p)\right]\left[1 - F_3(p)\right],$$

which again is convex in p. We can verify that $\pi_1''(\bar{p}_3) < \pi_1(\bar{p}_2) = \alpha\bar{p}_1$. In addition, the condition on k_2' ensures that $\pi_1''(r_2) < \pi_1(\bar{p}_2)$. This is because $(r_2 - \bar{p}_3) - \bar{p}_3\ln(r_2/\bar{p}_3) = k_2'$ by substituting F_2 into the definition of r_2, which implies that r_2 increases with k_2'. When $k_2' < (\gamma^* - \bar{p}_3) - \bar{p}_3\ln(\gamma^*/\bar{p}_3)$, $r_2 < \gamma^*$. Notice that γ^* solves the equation $\pi_1''(\gamma^*) = \alpha\bar{p}_1$. Hence, $\pi_1''(r_2) < \pi_1(\bar{p}_2)$, and thus $\pi_1''(p) < \pi_1(\bar{p}_2)$ for $\forall p \in [\bar{p}_3, r_2]$. Therefore, firm 1 has no profitable deviation by underpricing. In addition, for all firms, charging any price greater than $\bar{p}_1$ leads to zero profit, and charging any price below $\bar{p}_3$ leads to lower profit.

Altogether, all firms achieve constant profit within their given price supports and no firm has profitable deviation outside their price supports. Therefore, given consumers' search strategies, firms' pricing strategies form an equilibrium.

Next, we show that given firms' pricing strategies, the search strategies of nonshoppers listed in Table 1 are rational. Consider type 1 consumers' strategies. When making decision $d(z, \{1\})$, if $z \le r_1$, by Equation (6), the expected gain equals

$$EG\left(1; z, \{1\}\right) = \int_{\bar{p}_2}^{z} \left(z - p\right) dF_1\left(p\right) - k_1 \le 0$$

because r_1 is defined as

$$\int_{\bar{p}_2}^{r_1} \left(r_1 - p\right) dF_1\left(p\right) = k_1$$

and

$$\int_{\bar{p}_2}^{z} \left(z - p\right) dF_1\left(p\right)$$

is increasing in z. $d(z, \{2\})$ is to stop when $z \le \bar{p}_1$ because, by the definition of $\bar{p}_1$,

$$EG\left(2; \bar{p}_1, \{2\}\right) = \int_{\bar{p}_3}^{\bar{p}_1} \left(\bar{p}_1 - p\right) dF_2\left(p\right) - k_2$$

$$= \left[1 + \frac{\alpha \ln \alpha}{1 + \alpha\left(1 - \alpha\right)} - \frac{\ln\left(1 + \alpha\left(1 - \alpha\right)\right)}{1 - \alpha}\right] \bar{p}_1 - k_2 \le 0.$$

Similarly, for $d(z, \{3\})$, by the condition

$$k_3 \ge k_3' > \left[1 + \frac{\alpha \ln \alpha}{\left(1 - \alpha\right)\left[1 + \alpha\left(1 - \alpha\right)\right]}\right] \bar{p}_1,$$

we can check that

$$EG\left(3; \bar{p}_1, \{3\}\right) = \int_{\bar{p}_3}^{\bar{p}_2} \left(\bar{p}_1 - p\right) dF_3\left(p\right) - k_3 = \left[1 + \frac{\alpha \ln \alpha}{\left(1 - \alpha\right)\left[1 + \alpha\left(1 - \alpha\right)\right]}\right] \bar{p}_1 - k_3 < 0.$$

For $d(z, \{2,3\})$, notice that for any $z \le 1$, by Equation (6),

$$EG\left(2; z, \{2,3\}\right) = \int_{\bar{p}_3}^{z} \left(z - p\right) dF_2\left(p\right) - k_2 = EG\left(2; z, \{2\}\right).$$

The reason is that the third term on right-hand side of Equation (6) equals zero because the price p learned from the second firm is less than $\bar{p}_1$; thus, the lowest price $z' = \min\{z, p\} \le \bar{p}_1$, and the decision $d(z', \{3\})$ is to stop. By similar arguments, $EG(3; z, \{2, 3\}) = EG(3; z, \{3\})$. Therefore, comparing $EG(2; z, \{2, 3\})$ and $EG(3; z, \{2, 3\})$ is equivalent to comparing $EG(2; z, \{2\})$ and $EG(3; z, \{3\})$. Notice that the condition

$$k_3 - k_2 > \bar{p}_3 \frac{\alpha}{1-\alpha} \ln \alpha + \frac{\bar{p}_1}{1-\alpha} \ln\left(1+\alpha\left(1-\alpha\right)\right)$$

ensures that $EG(2;z,\{2\}) > EG(3;z,\{3\})$ for $\forall z \in [\bar{p}_1, 1]$. Therefore, if $z > \bar{p}_1$, it is optimal to inspect the second position; if $z \le \bar{p}_1$, it is optimal to stop because both $EG(2;z,\{2\})$ and $EG(3;z,\{3\})$ are nonpositive. For the decision $d(z,\{1,3\})$, we can similarly show that $EG(1;z,\{1,3\}) = EG(1;z,\{1\})$ and $EG(3;z,\{1,3\}) = EG(3;z,\{3\})$. Notice that the condition

$$k_1 < \frac{\alpha\left(1-\alpha\right) - \ln\left(1+\alpha\left(1-\alpha\right)\right)}{1+\alpha\left(1-\alpha\right)} \bar{p}_1$$

ensures that $EG(1;\bar{p}_1,\{1\}) > 0$ and $\bar{p}_2 < r_1 < \bar{p}_1$ by the definition of r_1. Recall that $EG(3;z,\{3\}) < 0$ if $z \le \bar{p}_1$. We can also conclude that for $\forall z \ge \bar{p}_1$, $EG(1;z,\{1\}) > EG(3;z,\{3\})$. Therefore, the decision $d(z,\{1,3\})$ listed in Table 1 is rational. For $d(z,\{1,2\})$, when $z \le \bar{p}_2$, $EG(1;z,\{1,2\}) = EG(1;z,\{1\}) < 0$ and $EG(2;z,\{1,2\}) = EG(2;z,\{2\}) < 0$, and thus to stop is optimal. Finally, we need to check the first-step decision $d(1,\{1,2,3\})$. We need to compare $EG(1;1,\{1,2,3\})$, $EG(2;1,\{1,2,3\})$, and $EG(3;1,\{1,2,3\})$. Based on the subsequent strategies, we can conclude that $EG(1;1,\{1,2,3\}) = EG(1;1,\{1\}) > 0$, which indicates that to enter the market is always rational. Also, $EG(3;1,\{1,2,3\}) = EG(3;1,\{3\})$, and we have $EG(1;1,\{1,2,3\}) > EG(3;1,\{1,2,3\})$ because $EG(1;1,\{1\}) > EG(3;1,\{3\})$, as is already shown. We can write the expected gain of first inspecting the second position as

$$EG\left(2;1,\{1,2,3\}\right) = \int_{\bar{p}_3}^{\bar{p}_1}\left(1-p\right)dF_2\left(p\right) - k_2 + \int_{r_1}^{\bar{p}_1}\left[\int_{\bar{p}_2}^{p}\left(p-x\right)dF_1\left(x\right) - k_1\right]dF_2\left(p\right)$$

$$< \left(1-\bar{p}_1\right) + \left[\int_{\bar{p}_3}^{\bar{p}_1}\left(\bar{p}_1 - p\right)dF_2\left(p\right) - k_2\right] + \int_{r_1}^{\bar{p}_1}\left[\int_{\bar{p}_2}^{\bar{p}_1}\left(\bar{p}_1 - x\right)dF_1\left(x\right) - k_1\right]dF_2\left(p\right)$$

$$< \left(1-\bar{p}_1\right) + 0 + \left[\int_{\bar{p}_2}^{\bar{p}_1}\left(\bar{p}_1 - x\right)dF_1\left(x\right) - k_1\right] = EG\left(1;1,\{1,2,3\}\right).$$

Therefore, it is optimal to start searching from the first position rather than the second or the third. In a similar manner, we can check that all strategies of the type 2 consumers listed in Table 1 are rational.

Altogether, under the given parametric conditions, the strategy profile specified in Proposition 5 is a rational-expectations equilibrium.