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This article studies how to endogenously assess the value of a "superior" advertising position in the price competition and examines the resulting location competition outcomes and price dispersion patterns. The authors consider a game-theoretic model in which firms compete for advertising positions and then compete in price for customers in a product market. Firms differ in their competence, and positions are differentiated in their prominence, which reflects consumers' online search behavior. They find that when endogenously evaluated within the product market competition, a prominent advertising position might not always be desirable for a firm with competitive advantage, even if it is cost-free. The profitability of a prominent advertising position depends on the trade-off between the extra demand from winning the position and the higher equilibrium prices when the weaker competitor wins it. Furthermore, the authors show that the bidding outcome might not align with the relative competitive strength, and an advantaged firm might not be able to win the prominent position even when it values that position. They derive two-dimensional equilibrium price dispersion with the realized prices at the same position varying and the expected prices differing across different positions. They find that the expected price in the prominent position might not always be higher, implying that an expensive location does not necessarily lead to expensive products.

Keywords: price competition, endogenous valuation, search advertising, online search, price dispersion, bidding incentive

Price Competition and Endogenous Valuation in Search Advertising

Search advertising, in which advertisers bid to be listed alongside search results or content pages for specific keywords, has been recognized as a successful revolution of the

traditional online and offline advertising. According to a recent industry survey conducted by the Interactive Advertising Bureau (IAB) and PricewaterhouseCoopers (PwC) (IAB and PwC 2011), Internet advertising revenues in the United States totaled $26.04 billion for 2010, 14.9% higher than 2009. Internet advertising surpassed newspapers to become the second largest advertising medium, next only to television, and is predicted to become the leading advertising medium in just a few years. Inside this prosperity, search advertising undoubtedly plays a leading role. Search-related advertising accounts for nearly half of the total Internet advertising revenue, outperforms all its predecessors (e.g., display advertisements, e-mail advertisements), and continues to grow rapidly.

While spending millions of dollars on search advertising, marketing managers inevitably wonder whether a seemingly attractive advertising position is indeed worth pursuing, how much value a premium sponsored slot creates for them, and how to allocate their marketing spending optimally. Should they bid aggressively to win a good position and charge a price premium to compensate for the spending,

or should they save money in the bidding and capture demand with price discounts through sales and promotions? From consumers' perspective, a typical question is where to find the best deal. Will the price from a prominent advertising slot be higher or lower than those from the less prominent positions? This study is intended to answer these questions.

A crucial step leading to the proper evaluation of sponsored advertising slots is to investigate their values endogenously in the product market competition, which can be easily neglected in industrial practice and academic research. Prominent positions (e.g., advertising slots listed on the top of a web page or highlighted in special color with extra space) usually generate high click-through rates and thus are commonly believed to be desirable. The existing literature on position auction mechanism design (e.g., Athey and Ellison 2008; Liu, Chen, and Whinston 2010), which carefully studies how advertisers shade their bids strategically in the bidding competition, usually assumes the value of per-advertisement-click as exogenously given and fixed. Under such an assumption, a prominent advertising slot that attracts more click-throughs naturally creates greater value for advertisers. Nevertheless, whether these clicks lead to final conversion also depends on the pricing of the product. In other words, the value of advertising slots is not realized without considering the price competition outcome. In this study, we suggest that the true value of a particular advertising slot to a particular advertiser should be understood endogenously in the price competition facing that advertiser. We show that, depending on the competitive situation, a more prominent slot may or may not be more valuable, even if it is cost-free. Instead of emphasizing the strategic bidding details, we focus on the value of advertising positions to advertisers and examine advertisers' willingness to pay for a prominent slot. We also analyze the resulting price dispersion associated with the bidding outcome. To the best of our knowledge, we are among the first to integrate price competition with bidding competition in the search advertising setting.

Like the advertising model itself, the price and bidding competition in search advertising is different by its nature from the traditional pricing and advertising topics in marketing and economics literature. There are two major distinctions: the unique features of online consumer search behavior and the asymmetric nature embedded in the competition for exclusive advertising resources.

Compared with traditional offline searching, consumers' online search behavior exhibits unique features: (1) a commonly observed search ordering and (2) highly diversified search costs for consumers. The first feature originates from the organization of advertisements in search engine results pages. The common format is that the sponsored links are listed in the right column alongside the organic search results, one after another from the top downward. Because of the reading habits and eye-movement pattern of most human beings, consumers usually process the information following the order of the list, from the top downward. Therefore, in general, consumers first pay attention to the top advertising slot of the sponsored list, and then the next, and so on, and some stop searching before reaching the bottom.[1]

The arrangement of advertisements and the resulting ordering of the search creates a large prominence difference among advertising slots with different ranks. Both online traffic statistics and empirical studies based on clickstream data have shown that the click-through rate attracted by the top link on a web page is, in general, the highest, and it decreases significantly from the top downward (e.g., Ansari and Mela 2003; Brooks 2004; Ghose and Yang 2009).

The second feature of online search behavior is related to the advance of information technology, which greatly facilitates informational searches. The physical cost to sample a product and get a price quote from a store, which would otherwise be a nonnegligible expense with necessary travel to the store, is now only a matter of several mouse clicks. In addition, some consumers derive hedonic utility from shopping online (e.g., Childers et al. 2001): They enjoy the process of searching different places, comparing prices, and finding the best deal, evidenced by those who spend hours surfing the web to shop. Altogether, with the flourishing of the Internet and online search engines, there arises a certain portion of consumers who have a nonpositive (zero or even negative) net search cost. We call them "shoppers." However, not everyone purchasing online has such luxury. The convenience of e-commerce brings many people with stringent time constraints, whose only goal is to find the product with a minimum amount of time spent. In addition, the information overload the Internet engenders and the extra skills needed to accomplish computer-based searches add to the cost for some online consumers. Therefore, there also exist a certain number of consumers who have a positive search cost, whom we call "nonshoppers."[2]

In addition to the distinctive features of online search behavior, the exclusiveness of the advertising resource and the asymmetry inherent in the bidding competition distinguish our study from existing ones. Traditional advertising technology allows advertisers to choose their advertising levels independently, which makes the competition outcome less sensitive to the asymmetry among firms' competence. In light of this feature, beginning with Butters (1977), the classical economics models of price advertising unanimously consider symmetric competition among advertisers and derive equilibrium outcomes in which all firms choose the same advertising level and adopt symmetric pricing strategy (e.g., Stahl 1994; Stegeman 1991). In contrast, in search advertising, the prominent advertising slot is sold by auction and by its nature is exclusive: Only one firm can win the most prominent position. Therefore, a slight difference in firms' competence could lead to a large difference between winning and losing the best business location. This type of advertising thus demands that we capture even a small difference in firms' competence and tease out the bidding result explicitly. Asymmetric competition involves firms that play different strategies in equilibrium, which brings challenging yet intriguing aspects into the analysis, such as the determination of the winning bid and the com-

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[1]Hoque and Lohse (1999) use experimental data showing that, compared with traditional paper media, consumers are more likely to pay more attention to the advertisements near the beginning of the heading in online directories.

[2]Early consumer research has shown that a consumer's search effort is determined by various factors, such as time availability, purchase involvement, and attitudes toward shopping, and consumers do not always search thoroughly, even when purchasing expensive items (Beatty and Smith 1987). A recent empirical investigation shows that online shoppers tend to search few sites on average (Johnson et al. 2004).

parison of the price expectation. This study is among the few that handle asymmetric competition under the price advertising setting.

To study the interaction between pricing and bidding competition under search advertising, we set up a game-theoretic model involving asymmetric competition among firms and capturing the aforementioned features of online consumer search behavior. Firms first compete for advertising slots by auction and then compete for customers in a product market. Firms differ in their competence, which is represented by production cost. Positions are differentiated in their prominence, which reflects the ordering of the list. Consumers start searching from the prominent position. The shoppers conduct a thorough search, while nonshoppers stop searching when their reserve price is satisfied. Thus, we provide an integrated framework to endogenously investigate the value of advertising slots in the context of both bidding and pricing competition, taking into account firms' relative competitive strength and consumers' online search behaviors.

On the basis of this framework, we show that a prominent advertising slot might or might not be desirable. In some cases, even without any extra cost, the prominent position is not desirable for the firm that has a competitive advantage, which underscores our previous argument that whether a seemingly prominent position is indeed superior should be determined endogenously, taking competitors' responses in pricing into consideration. The major trade-off facing the firm with a competitive advantage is exploiting the extra demand when winning the prominent position versus charging high prices when letting the weak competitor win the better position. We also find that the bidding outcome might not always be in favor of the firm with a competitive advantage. In some scenarios, the disadvantaged firm might be able to outbid its competitor and win the prominent position.

By analyzing the equilibrium pricing, we identify a unique pattern of two-dimensional price dispersion: First, we show that the presence of shoppers makes any static pricing unstable, and in equilibrium, the *realized* price from the same position varies. Second, because of the common search ordering and the resulting location prominence difference, firms at different positions adopt different pricing strategies, and the *expected* prices from different positions differ. We reveal that the price expectation from a prominent position might not be higher, which is somewhat different from the common wisdom that an expensive location comes with expensive products.

It is worth noting that in addition to online search advertising, we formulate and analyze the whole problem in a way that the model and its implications are also applicable to other settings involving price promotion and location acquisition, from storefront location competition to slotting space allocation in retail stores, as we discuss in more detail in the "Conclusion" section of this article. Along this line, there is rich literature on price promotion and geographical location competition. Varian (1980) studies symmetric mixed-strategy pricing when some "informed" consumers are fully aware of the prices from all firms and the others are "uninformed." Narasimhan (1988) and Raju, Srinivasan, and Lal (1990) consider duopolistic price competition when some consumers are "loyal" to one brand. These works consider only the price competition and take the information

structure or the market segmentation as exogenously given. In contrast, we consider both price and bidding competition so that the "guaranteed" demand is acquired endogenously. In addition, the location advantage is exclusive, so that the allocation outcome reflects the subtle interactions embedded in the asymmetric competition, and firms adopt asymmetric pricing strategies upon winning. Other literature has focused on geographical location competition. For example, Dudey (1990) shows that sellers choose to cluster together when buyers incur higher search costs across different locations. In contrast, we explicitly model prominence difference among locations and consider exclusive location choice decisions.

We organize the rest of the article as follows: In the next section, we begin with a baseline model to capture the essence of our interest to derive neat results and clear insights. We consider two heterogeneous firms competing for a prominent advertising slot to sell products to consumers. We temporarily leave consumers' search behavior exogenous. The following section details the analysis and derives results from the baseline model. Then, we endogenize consumers' search strategies. First, we endogenize consumers' choice of search ordering and allow them to deviate from the presumed order, and second, we endogenize consumers' sequential search decision and let them strategically decide whether to continue or stop searching. We show that the qualitative results derived from the baseline model stay the same. In the "Extension and Discussion" section, we further extend the baseline model along various directions. We show that the main results continue to hold when search advertising is not the only information channel, when consumers have heterogeneous preferences over firms' products, and when there are more than two firms competing. In addition, we provide some supportive observations that are consistent with our modeling results. We conclude with a discussion of the managerial implications.

THE BASELINE MODEL

Two firms compete for a prominent advertising position by auction. The winning firm is placed in the prominent position, which is called the first position, or Position 1. The other firm stays at a less prominent position, which is called the second position, or Position 2. The prominent advertising position can be interpreted as the top-sponsored advertising slot listed on a search engine results page. Depending on whether they win the prominent position, the firms compete for consumers by setting different prices. The firms are selling a homogeneous product. The homogeneity assumption is partially motivated by the high degree of standardization and digitization of products or services on the Internet. More fundamentally, suppressing heterogeneity among products enables us to see clearly how locational effect (and essentially consumers' online search behaviors) alone generates a significant level of price dispersion (for the same product). Relaxing this assumption to consider heterogeneous consumer preferences does not change the main insights, as we show subsequently. The firms are differentiated in their competence, which is represented by the marginal production cost. The firm with competitive advantage, termed "high type" (H), has a lower marginal production cost c_1, while the firm with competitive disadvantage, termed "low type" (L), has a higher marginal production

cost c_2. Without loss of generality, we normalize c_1 to 0 and denote c_2 as c, where c > 0.

There is a continuum of consumers of measure 1. Each consumer has a unit demand of the product. Consumers obtain price information about the product by sampling the advertising slot(s) (e.g., clicking the sponsored link[s]). We assume that all consumers start sampling products from the prominent position. Among them, α (0 < α <1) are nonshoppers who sample the first position only. The other 1 – α are shoppers who sample both positions. Here, we leave consumers' search behavior exogenous to avoid unnecessary technical complexity in the baseline model, while still capturing the characteristics of consumer online search behavior (i.e., a commonly observed search ordering and highly diversified consumers' search costs). We later relax the ordering assumption to allow consumers to choose the search ordering strategically, not necessarily starting from the prominent position. In addition, we endogenize consumers' sequential search decisions, showing that the equilibrium outcome coincides with the exogenous assumptions. Again, to keep the baseline model simple and to avoid unnecessary distraction from the demand factor, we assume that all consumers have the same willingness to pay for the product, w, where w > c. Relaxing this assumption to a case with consumers of heterogeneous willingness to pay does not change the main results (as we detail in Web Appendix A, http://www.marketingpower.com/jmrjune11). Consumers buy the product when the price is no greater than w. Consumers who sample both positions have perfect recall on price and make the purchase from the firm with the lower price; if the price is the same for both, they randomly pick one, with equal probability.

The determination of the auction outcome is based on a score that equals a unit price bid times a weighting factor. Each firm submits a per-visit (or per-click) bid b_i (i $\in$ {H,L}). The weighting factor ω_i equals the expected visits (clicks) if firm i is placed in the first position. In the baseline model, the weighting factors simply equal 1 for both firms. In the extensions, ω_i may take different values depending on different settings. The firm with the highest score $s_i = \omega_i b_i$ wins the first position and pays on a per-visit basis such that the per-visit payment generates a score equal to the second highest score $s_{i'} = \omega_{i'} b_{i'}$; that is, the winning firm i pays $\omega_{i'} b_{i'}/\omega_i$ per visit ({i, i'} = {H, L}). The firm that does not win the first position stays in the second one and pays a reserve price, which we assume to be zero for simplicity.[3]

We consider a two-stage game. In the first stage, the two firms decide their bidding strategies and are placed in the corresponding positions according to the auction rule. In the second stage, the firms price their product, and consumers sample the position(s) and make purchase decisions.[4] Considering the transparency of business information within the

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[3]Imposing a positive reserve price may involve issues of firms' entry decision; however, it does not change the main results.

[4]The chosen timeline of the game enables us to explicitly examine the values of different advertising positions. It is also a natural timeline applicable to general contexts involving pricing and location acquisition. In fact, the equilibrium outcomes of bidding and pricing competition exhibit similar patterns under different timeline settings. For example, when bidding and pricing decisions are simultaneous or in a reverse order, in equilibrium, the low-type firm is likely to win the first position only if c and α are relatively small, and similar pricing strategies arise in equilibrium.

same industry and highly repetitive interaction in search advertising, we follow the common approach in the literature (e.g., Edelman, Ostrovsky, and Schwarz 2007; Varian 2007) and consider complete information structure (i.e., the game structure, auction rules, and all parameters are common knowledge to both firms).

MAIN RESULTS

Along the line of backward induction, we look for a subgame perfect equilibrium. We first consider the secondstage price competition given the winning positions and then study the bidding competition in the first stage. Because of the existence of shoppers, meaning that there exist a certain portion of consumers who search around, know all price information, and purchase from the firm offering a lower price, a slight price cut relative to the competitor can help a firm capture this portion of demand and thus results in a significant increase in the sales profit. As a result, any static prices from the two firms cannot be an equilibrium. In other words, there is no pure-strategy equilibrium in the second-stage price competition.

We next explore the mixed-strategy equilibrium. We use $F_i(p)$, i $\in$ {H, L}, to describe a firm's mixed strategy of pricing. Like regular cumulative distribution functions, $F_i(p)$ measures the probability that the firm will charge a price less than or equal to p.

Lemma 1: (a) Given that H wins the first position, the equilibrium mixed strategies of pricing in the second stage are

(1)
$$F_H(p) = \begin{cases} \dfrac{p - m}{p - c} & p \in [m,w) \\ 1 & p = w \end{cases}$$

$$F_L(p) = \begin{cases} \dfrac{p - m}{(1 - \alpha)p} & p \in [m,w) \\ 1 & p = w \end{cases},$$

where m = max{αw, c},[5] and (b) Given that L wins the first position, the equilibrium pricing strategies are

(2)
$$F_H(p) = \dfrac{p - c - \alpha(w - c)}{(1 - \alpha)(p - c)} \qquad p \in [c + \alpha(w - c), w]$$

$$F_L(p) = \begin{cases} \dfrac{p - c - \alpha(w - c)}{p} & p \in [c + \alpha(w - c), w) \\ 1 & p = w \end{cases}.$$

The Appendix details all proofs.

The preceding lemma describes an asymmetric mixed-strategy equilibrium in the second-stage price competition under different situations. Note that when H wins the first position and cost advantage is the dominating factor (specifically, c $\geq$ αw), H would play pure strategy and charge a competitive price equal to L's marginal cost, with the aim of gaining the entire market. Meanwhile, L mixes over prices, ensuring that H has no profitable deviation and earns zero profit itself. In this sense, this scenario is close to

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[5]To be rigorous, when m = c, we define the value of $F_H(p)$ at c as $\lim_{p \to c+} (p - c)/(p - c) = 1$.

a standard Bertrand competition with asymmetric production costs.[6]

In all other cases, both firms play mixed strategies and achieve positive expected profit. Similar to other asymmetric mixed-strategy equilibrium found under different settings (e.g., Amaldoss and Jain 2002), the equilibrium pricing strategies should satisfy the following properties. First, both firms' equilibrium price distributions have a common and continuous support such that $F_H(p)$ and $F_L(p)$ are strictly increasing on the common support $[\underline{p}, \overline{p}]$. Otherwise, pricing within those nonoverlapped ranges would lead to a suboptimal profit level. Second, there is no mass point in both firms' distributions on $[\underline{p}, \overline{p}]$ such that F_H and F_L are continuous on $[\underline{p}, \overline{p}]$. This is because a mass point in one firm's price distribution would result in a downward jump of the other firm's expected demand at that point and consequently lower profit levels in a contiguous region on the right side of that point. In addition, by similar arguments, at most one firm may have a mass point at $\overline{p}$. These properties ensure that the equilibrium identified in the preceding lemma is a unique equilibrium in the second-stage pricing game.

Given that H wins the first position and $\alpha w > c$, the highest price that H can charge is consumers' willingness to pay w, which ensures H a profit level of αw by exploiting all the surplus of its guaranteed demand α. In light of this line, H will never take any price below αw, because charging a lower price would certainly lead to a profit lower than αw. This explains the price range $[\alpha w, w]$. Similarly, in the case in which L wins the first position, L can earn at least $\alpha(w - c)$ by charging w such that it will never take any price lower than $c + \alpha(w - c)$.

Note that when adopting mixed strategies, firms earn the same expected profit across the price range involved. Therefore, firms' expected profits can be specified by examining the expected profits when firms charge the lowest equilibrium prices, as Table 1 summarizes, in which π_i^j denotes firm i's expected sales profit in position j.

One question of particular interest is whether a prominent position is always desirable—in other words, whether a firm can achieve higher expected profit in the prominent position than otherwise. The following proposition shows that a seemingly prominent position is not always desirable.

P$_1$ (endogenous valuation): For the low-type firm, staying in the first position always brings higher profit than staying in the second one; however, for the high-type firm, when $\alpha^2/(1 - \alpha)^2 < c/w < (1 - \alpha)/(2 - \alpha)$, the second position is more profitable.

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[6]When there is a finite minimum money increment ε, L pricing c and H pricing $c - \varepsilon$ can be a pure-strategy equilibrium in this scenario. Here, we follow the convention and treat money as infinitely divisible. Both cases lead to the same results.

Table 1
FIRMS' EXPECTED PROFITS IN DIFFERENT SITUATIONS

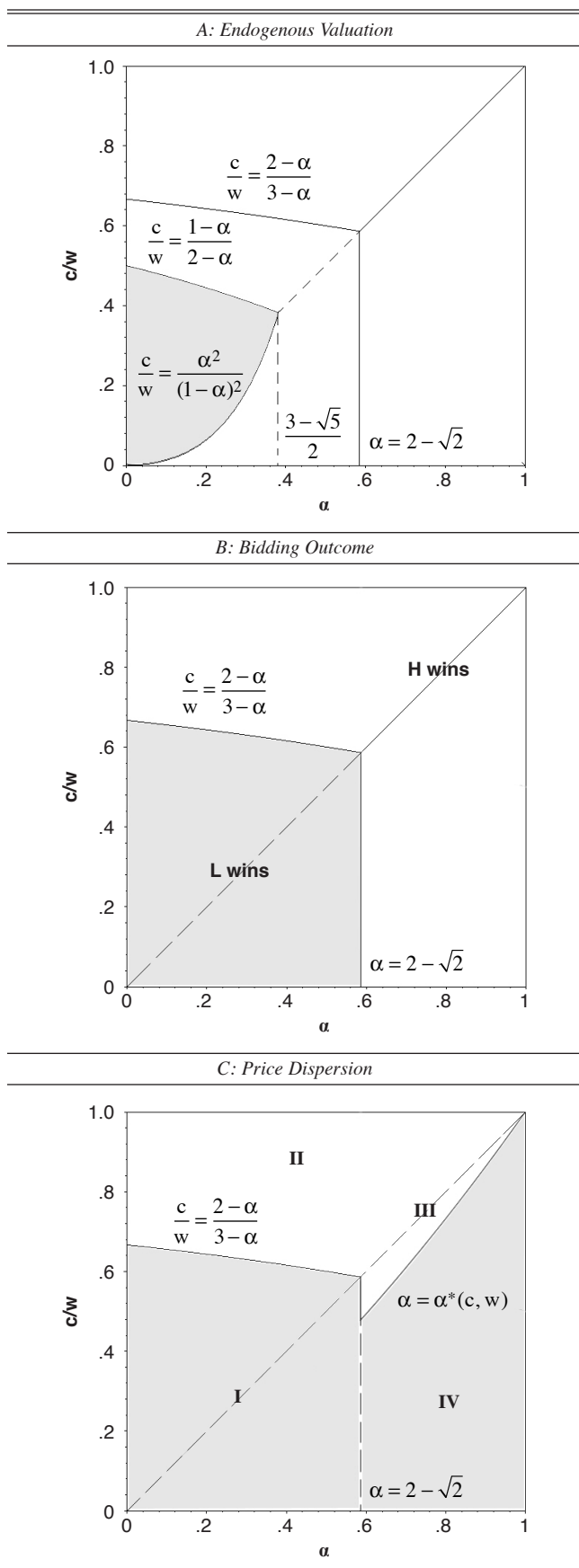|	When H wins	When L wins
H's expected profit	$\pi_H^1 = m$	$\pi_H^2 = (1 - \alpha)[c + \alpha(w - c)]$
L's expected profit	$\pi_L^2 = (1 - \alpha)(m - c)$	$\pi_L^1 = \alpha(w - c)$

Notes: $m = \max\{\alpha w, c\}$.

As Figure 1, Panel A, illustrates, in the shadowed region, H can achieve higher expected profit in the less prominent position than in the prominent one. This surprising result reveals the nature of price competition and captures the trade-off the high-type firm faces between exploiting the prominent location and benefiting from its lower cost. Intuitively, after winning the prominent position, the low-type firm exploits it thoroughly by charging a relatively high price because it is at a cost disadvantage and the prominent position ensures a guaranteed demand. For this reason, although staying in the second position and receiving less attention, the high-type firm may earn reasonable revenue by obtaining most of the residual demand at a relatively high price. However, when it is at the prominent position, the high-type firm may face fierce competition from the low-type firm on the residual demand because the low-type firm is desperate to appeal to some consumers. Therefore, when the loss of demand on the less prominent position is not too high (i.e., α is small), staying in the second position could be more profitable for the high-type firm as long as the benefit from the cost difference is not too low (i.e., $c/w > \alpha^2/(1 - \alpha)^2$). Nevertheless, the cost advantage cannot be too high, either; otherwise, the high-type firm could enjoy the profit from the prominent position by excluding its opponent and occupying the entire market, which far exceeds what it can earn in a less prominent position. This explains $c/w < (1 - \alpha)/(2 - \alpha)$. In brief, whether the prominent position is worth pursuing for the firm with competitive advantage essentially depends on the trade-off between capturing the nonshoppers (when winning the advantageous position) and increasing the price premium charged to the shoppers (when letting the weaker opponent win).

P$_1$ illustrates a scenario in which a prominent advertising position, though generating more clicks, may have less value than a less prominent one for the high-type firm, which indicates that the per-click value of the prominent position could be significantly different from that of the less prominent one. The result underscores that rather than blindly pursuing a prominent advertising position with a high click-through rate, firms should evaluate an advertising position endogenously within the actual competitive environment, taking into consideration firms' relative competence and consumers' search patterns.

Meanwhile, in many cases, the prominent position does have its value, and thus, both firms compete for the position. We next consider the bidding competition in the first stage. We first derive firms' equilibrium bidding in a general way to show clearly how the equilibrium bidding depends on the model parameters. Recall that the firm with the higher score $s_i = \omega_i b_i$ wins, where b_i is the per-click bid and ω_i is the weighting factor, and $i \in \{H, L\}$. In addition, we denote the expected clicks when firm i stays in the first position as λ_i. Therefore, if $\omega_i b_i > \omega_{i'} b_{i'}$, firm i wins the first position, pays $\omega_{i'} b_{i'}/\omega_i$ per click, and achieves a net expected profit level equal to $\pi_i^1 - \lambda_i \omega_{i'} b_{i'}/\omega_i$; otherwise, firm i stays at the second position with net expected profit π_i^2. Suppose $\pi_i^1 > \pi_i^2$; in that case, it is profitable for firm i to outbid its rival (i.e., to bid $b_i > \omega_{i'} b_{i'}/\omega_i$) if and only if $\lambda_i \omega_{i'} b_{i'}/\omega_i < \pi_i^1 - \pi_i^2$. Therefore, bidding $b_i = (\pi_i^1 - \pi_i^2)/\lambda_i$ is firm i's weakly dominant strategy. Thus, we show that in equilibrium, independent of the weighting factors ω_i, firms bid in such a way that the total willingness to pay ($b_i \lambda_i$) equals $\pi_i^1 - \pi_i^2$. Moreover,

Figure 1
EQUILIBRIUM OUTCOMES: THE BASELINE MODEL

A: Endogenous Valuation

$$\frac{c}{w} = \frac{2-\alpha}{3-\alpha}$$

$$\frac{c}{w} = \frac{1-\alpha}{2-\alpha}$$

$$\frac{c}{w} = \frac{\alpha^2}{(1-\alpha)^2}$$

$$\frac{3-\sqrt{5}}{2}$$

$$\alpha = 2 - \sqrt{2}$$

B: Bidding Outcome

H wins

$$\frac{c}{w} = \frac{2-\alpha}{3-\alpha}$$

L wins

$$\alpha = 2 - \sqrt{2}$$

C: Price Dispersion

II

$$\frac{c}{w} = \frac{2-\alpha}{3-\alpha}$$

III

$$\alpha = \alpha^*(c, w)$$

I

IV

$$\alpha = 2 - \sqrt{2}$$

because we let the weighting factor ω_i equal the expected clicks λ_i, which is commonly believed to be the major search engines' practices, the score $s_i = \omega_i(\pi_i^1 - \pi_i^2)/\lambda_i = \pi_i^1 - \pi_i^2$, and thus, advertisers are ranked essentially according to their endogenous valuation of the prominent position. Here, in the baseline model, the situation is simpler because $\omega_i = \lambda_i = 1$ for i = H, L. As a result, $b_i = \max\{\pi_i^1 - \pi_i^2, 0\}$, and the firm with a higher bid wins the position. By comparing b_H and b_L under different situations, we can conclude the equilibrium bidding outcome, as P_2 summarizes and Figure 1, Panel B, illustrates.

> P_2 (bidding outcome): (a) When $\alpha < 2 - \sqrt{2}$ and $c/w < (2 - \alpha)/(3 - \alpha)$, the low-type firm wins the first position, and (b) when $\alpha > 2 - \sqrt{2}$ or $c/w > (2 - \alpha)/(3 - \alpha)$, the high-type firm outbids its rival.

P_2 reveals an asymmetric equilibrium in the bidding for the prominent position. It provides the rationale for firms to determine their spending on location competition according to their relative competitive strength and consumers' search behavior, to better position themselves in the marketing campaign. This proposition implies that when either the competence difference or prominence difference is a dominating factor, the firm with a competitive advantage should compete aggressively to acquire the prominent position.

The intuition is as follows. A firm with a cost advantage can easily outperform its competitor in the price competition and garner most of the market share. Therefore, a prominent location is worth pursuing only if a significant difference in prominence exists; otherwise, as long as a certain portion of consumers will visit both sites, the firm can stay in a less prominent position, still win most of the residual demand, and attain a satisfactory profit level. This explains why only a high-alpha position motivates the high-type firm to bid aggressively. In contrast, a firm with a cost disadvantage suffers significantly when its cost disadvantage is large, which greatly diminishes the profitability of staying in a prominent position. Therefore, the prominent position is more desirable to the low-type firm when c is relatively small.

An interesting aspect revealed by this proposition is that the bidding outcome may not always be in favor of the firm that has a competitive advantage. As a result of endogenous consideration in product market competition, the firm's competitive strength may not align with the competition result. In some scenarios, although the high-type firm values the prominent position and wants to win, it cannot afford to bid as high as the low-type firm. This result is in contrast to the efficient allocation property of the auctions (i.e., high-type bidder wins the prominent position) revealed in the framework with exogenous bidder valuations (e.g., Liu, Chen, and Whinston 2010).

It is worth mentioning that the preceding equilibrium outcome incorporates the extreme cases, namely, $c = 0$, $\alpha = 0$, or both. When the cost difference disappears ($c = 0$), meaning that firms are homogeneous, the model reduces to the commonly seen symmetric competition model, in which symmetric bidding and pricing strategies arise in equilibrium. According to Table 1, $\pi_H^1 - \pi_H^2$ and $\pi_L^1 - \pi_L^2$ will become the same. Therefore, they will bid equally at $b = \alpha^2 w$ and achieve the same net profit $(1 - \alpha)\alpha w$, regardless of whether they win the first position. Moreover, Equations 1 and 2

give the same equilibrium pricing strategy of H and L when in the same position. Likewise, when the location prominence difference vanishes ($\alpha = 0$), meaning that all consumers visit both positions, it reduces to the typical Bertrand competition. Because the two positions thus become the same, neither firm is willing to spend any money in the bidding. According to Table 1, $\pi_H^1 = \pi_H^2 = c$ and $\pi_L^1 = \pi_L^2 = 0$. Regardless of their positions, H always charges $p_H = c$, whereas L plays the same mixed strategy $F_L(p) = (p - c)/p$, according to Equations 1 and 2. If both $\alpha = 0$ and $c = 0$, we arrive at the trivial case in which both firms charge the competitive price $p = 0$ and gain zero profit.

Next, we examine the prices from different positions. Intuitively, because bidding is costly, the winner of the prominent position tends to charge a higher price to compensate itself, and thus, the expected price from the prominent position should be higher. Nevertheless, as we summarize in the following proposition, it might not always be the case:

P$_3$ (price dispersion): (a) When $\alpha < 2 - \sqrt{2}$ and $c/w < (2 - \alpha)/(3 - \alpha)$, the expected price from the first position is higher than that from the second one, and (b) when $\alpha > 2 - \sqrt{2}$ or $c/w > (2 - \alpha)/(3 - \alpha)$, if $\alpha > \alpha^*(c, w)$, the expected price from the first position is higher; if $\alpha < \alpha^*(c, w)$, the opposite is true, where $\alpha^*(c, w)$ is determined by

$$(3) \qquad (\alpha^* - c/w)\ln\frac{1 - c/w}{\alpha^* - c/w} + \alpha^* + \frac{\alpha^*}{1 - \alpha^*}\ln\alpha^* = 0.$$

It is noteworthy that the price expectation from the prominent position may be lower when the high-type firm has a significant cost advantage that is not overwhelmed by the prominence difference. In this case, the high-type firm wins the prominent position and would rather take advantage of its low cost to garner more market share through intense price cutting. As the unshadowed regions (Regions II and III) in Figure 1, Panel C, illustrate, Region II accounts for the case when the high-type firm plays pure strategy, and Region III accounts for the mixed-strategy case, in which the high-type firm puts the most mass on the price range close to the lower bound of the support, yielding a lower price expectation.

In all other cases, as can be expected, the prominent position winner reaps its location advantage by charging nonshoppers a higher price in general (shadowed region in Figure 1, Panel C). When the low-type firm wins the first position (Region I in Figure 1, Panel C), according to Equation 2, its price distribution first-order stochastically dominates that of the high-type firm.

Note that we derive two dimensions of price dispersion at the same time from the model. First, firms mix their prices in equilibrium, indicating that rather than charging one price deterministically, they price with uncertainty. Therefore, the realized price from the same position can vary over a certain range probabilistically. This feature coincides with the complexity and uncertainty in determining the final prices of products actually observed (e.g., different shipping and handling fees, various coupon discounts and cash rebates). Moreover, as Varian (1980) proposes, when considered over a long period, the mixed-strategy pricing can lead to price fluctuation over time (e.g., with occasional promotions, markups), which accounts for the temporal price dispersion.

Second, firms at different positions adopt different pricing strategies in equilibrium so that the expected prices from different positions differ, which accounts for the spatial price dispersion. Empirical evidence of online price dispersion in both dimensions has been well documented in the literature.[7]

It is worthwhile to pinpoint the driving forces of the unique two-dimensional price dispersion pattern derived here. The first dimension is the result of the presence of shoppers who always search around and thus make any static pricing unstable. The spatial dispersion originates from the search ordering and its resulting prominence difference. Because the majority of consumers observe a common search ordering and nonshoppers conduct limited search, a prominent position attains its prominence advantage by easily attracting consumers' attention and retaining a portion of them. Such an asymmetric prominence leads to different expected prices for different positions. Moreover, the two-dimensional dispersion is further enriched by the asymmetry in advertisers' competitive strength. Partially reflecting the bidding outcome, spatial dispersion can occur in one way or the other, depending on the competence difference (compared with the prominence difference).

Next, we present some comparative statics results on how model parameters affect advertisers' net profits (i.e., sales profit net of bidding cost), social welfare, and the revenue of the advertising provider (e.g., the search engines) in equilibrium. For advertisers' net profits, simple algebra shows that in equilibrium, the high-type firm always achieves higher net profit than the low-type firm, despite all the complexity of gain and loss in the bidding and price competition. This result simply implies that the cost advantage is indeed rewarding. Furthermore, Figure 2 illustrates how both firms' net profits change in α given different c, where the bold curves highlight the net profit of the prominent position winner. Note that neither firm's equilibrium net profit changes monotonically in α. The nonmonotonicity originates from two counteracting effects that α affects the sales profit at the second position: On the one hand, a larger α takes away more market share. On the other hand, a larger α leaves higher profit margins because the prominent position winner tends to charge a higher price. The winner must pay a total price equal to the competitor's profit difference between staying in the two positions, thus introducing such nonmonotonicity into the bidding cost and then the net profit. Note that even from the prominent position winner's perspective, a higher prominence advantage α may not necessarily lead to a higher equilibrium net profit.

The overall social welfare equals the sum of consumers' surplus, advertisers' net profit, and the advertising provider's revenue. Essentially, it equals the total consumer value realized from the consumption of the products minus firms' production costs. For example, when L wins the first position in equilibrium, we can write the expected social welfare as follows:

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[7]Smith and Brynjolfsson (2001) and Chen and Hitt (2002) show that significant levels of price dispersion exist online across different firms, even after control for various heterogeneities. Baye, Morgan, and Scholten (2004) find that the identities of the lowest-priced firms for various online products keep changing over time, which suggests a persistent level of temporal price dispersion (for a detailed literature review, see Pan, Ratchford, and Shankar 2004).

Figure 2
THE CHANGE OF ADVERTISERS' NET PROFITS (ξ) IN α

A: $w = 1, c = .2$

B: $w = 1, c = .5$

C: $w = 1, c = .8$
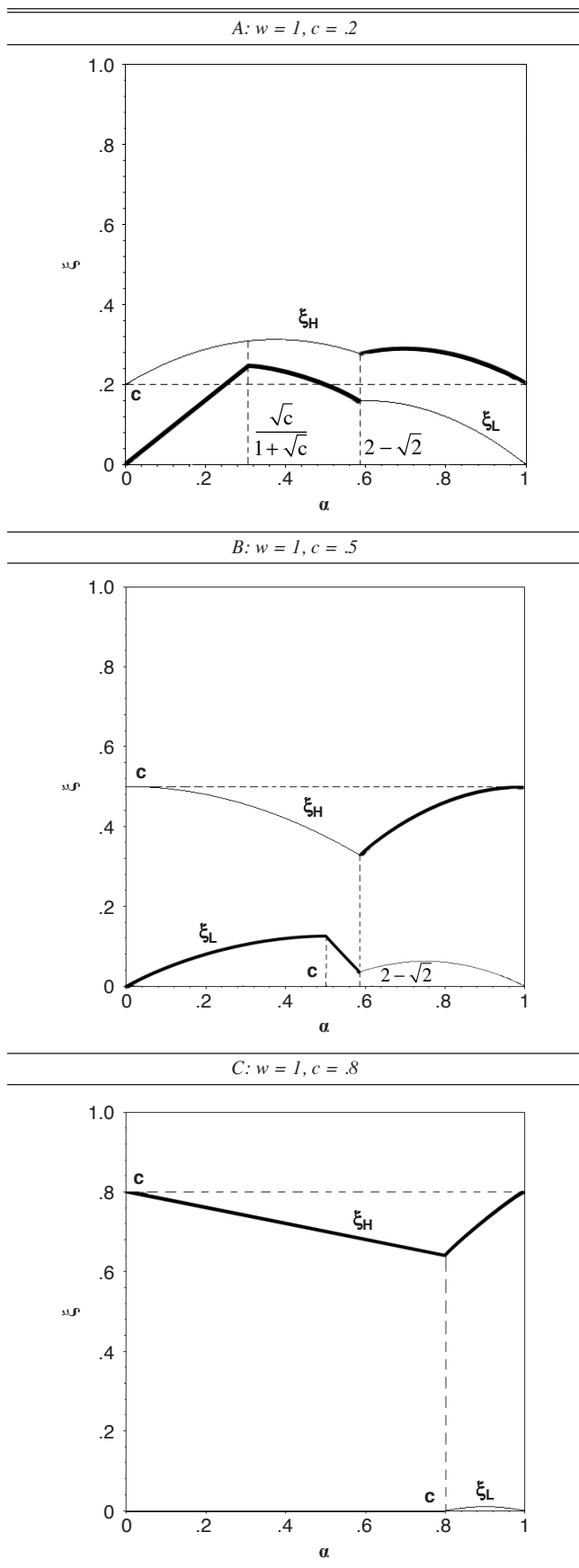
$$W = w - \left[\alpha + (1 - \alpha) \int_{\underline{p}}^{\bar{p}} F_L(p)\, dF_H(p) \right] c, \tag{4}$$

where the integral represents the probability of L's charging a lower price than H, which in turn measures L's expected sales to the shoppers. Figure 3 illustrates how the welfare changes in α and c in different scenarios. Note that the jumps depicted in both Panels A and B correspond to the points when the bidding outcomes reverse (H wins on the right-hand side of the jumps and L wins on the left). As the figure indicates, the V-shaped welfare curves indicate that when the winning (low-type) firm causes too much welfare loss, it is automatically replaced by its competitor. In this sense, in allocating the exclusive advertising resource, the auction mechanism serves as an auto-adjustment to prevent substantial welfare loss.

The search engine's revenue is determined by the winning firm's payment, which in turn is determined by the second highest bid. Thus, it also reflects the competitiveness of the bidding. Figure 4 depicts the level curves of the search engine's revenue. As the figure indicates, the revenue increases rapidly toward the right bottom corner. Small c means that the two firms are close to each other in terms of

Figure 3
THE CHANGES OF SOCIAL WELFARE IN α AND c

A: The Changes of Social Welfare in α ($w = 1$)

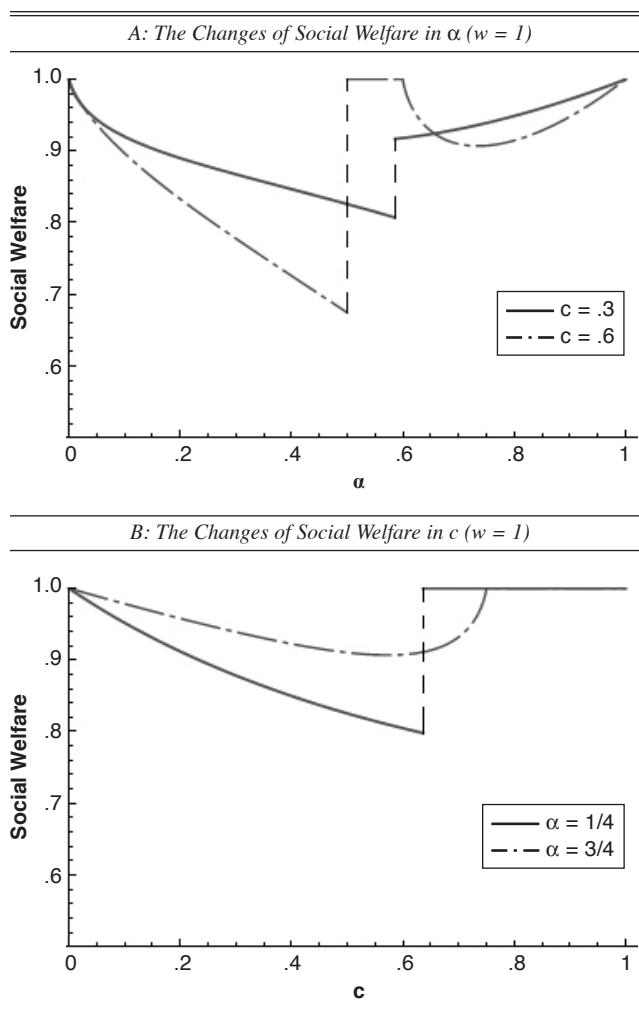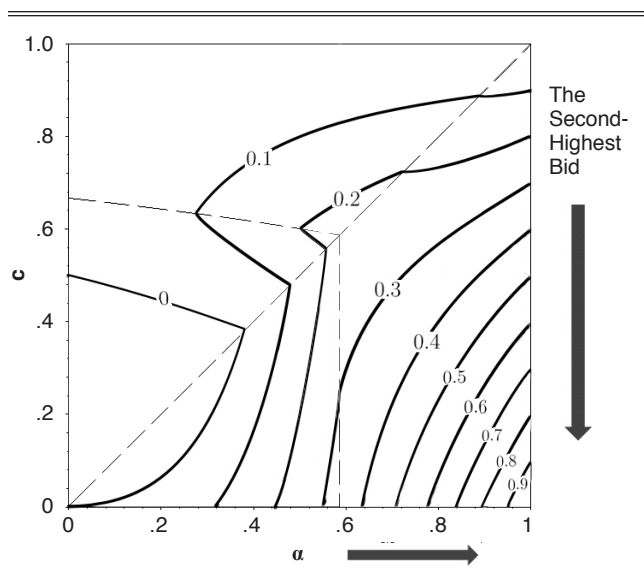B: The Changes of Social Welfare in c ($w = 1$)

Figure 4
THE LEVEL CURVES OF SEARCH ENGINE'S REVENUE

competitive strength. Meanwhile, a large α indicates a large difference between winning the prominent position and not. The combination of the two induces firms to compete intensively for the prominent position.

ENDOGENOUS CONSUMER SEARCH

In the baseline model, we leave consumers' search behaviors as exogenous to induce easily understood analysis, neat results, and clear insights, while avoiding too much technical complexity. In this section, we endogenize consumer search. In particular, we consider consumers' strategic choice of search ordering by allowing them to start searching from the position with a lower expected price; we also consider consumers' endogenous sequential search decision (i.e., whether to conduct another search) by allowing them to rationally assess the expected gain from an additional search. As the subsequent discussion shows, the qualitative results from the baseline model remain the same.

Strategic Choice of Ordering

Under the assumed consumer search ordering, the preceding discussion shows that spatial price dispersion does exist, and the expected price from the prominent position could be higher. Understanding this, some sophisticated consumers may anticipate firms' pricing strategies and start sampling from the position with the lower (expected) price instead of simply following the presumed search behavior. We now consider the case in which some consumers are sophisticated and strategically choose their search ordering. We continue to consider the diversification among consumers' search behavior: Shoppers and nonshoppers coexist. Note that for sophisticated shoppers, it actually does not matter in which position they begin. However, for the sophisticated nonshoppers, their rational behaviors may affect firms' decision and alter the competitive picture to some extent.

Following the framework of the baseline model, we continue to assume that α of the consumers are nonshoppers

who sample only once and $1 - \alpha$ are shoppers who sample both positions. Now we consider that among all the consumers, a portion of them, β ($0 < \beta < 1$), are sophisticated, and they can anticipate firms' strategies and start sampling from the position with the lower (expected) price. The rest $(1 - \beta)$ simply start sampling from the first position. In other words, $1 - \alpha$ of all the consumers sample both positions (it does not matter in which position they begin), $\alpha\beta$ of the consumers sample the position with the lower (expected) price only, and the rest $\alpha(1 - \beta)$ only consider the first position. In the first stage, firms decide their bidding strategies and obtain different positions according to the auction rules. In the second stage, sophisticated consumers observe the bidding outcome and decide in which position to begin, and meanwhile, firms price their product. Then, all consumers sample the position(s) and make purchase decisions. We continue to consider complete information so that the game structure is common knowledge to firms and consumers.

The strategy profile can be written as $\{b_i, F_i(\cdot; s_H, s_L), \sigma(s_H, s_L): i \in \{H, L\}\}$, where, as in the baseline model, b_i is firm i's per-click bid in the first stage and $F_i(\cdot; s_H, s_L)$ is the cumulative distribution function of firm i's pricing strategy in the second stage, contingent on the bidding outcome (i.e., the comparison of the bidding scores s_H and s_L). Here, we use $\sigma(s_H, s_L)$ to describe the strategy of those sophisticated consumers: Observing the outcome of the auction, they begin sampling from the first position with probability σ, and they begin from the second position with probability $1 - \sigma$, $0 \leq \sigma \leq 1$. For a strategy profile to be a subgame-perfect rational expectations equilibrium, it should satisfy the following two conditions: First, given the outcome of the bidding competition in the first stage, $\{F_i(\cdot; s_H, s_L), \sigma(s_H, s_L): i \in \{H, L\}\}$ must be a rational expectations equilibrium. Specifically, given the assigned positions and sophisticated consumers' strategy, the firms have no profitable deviation in their pricing strategies in the second stage. Meanwhile, sophisticated consumers are rational, which means that their belief about which position has a lower expected price is consistent with firms' equilibrium outcome; in other words, $\sigma = 1$ if $E(p^1) < E(p^2)$, $\sigma = 0$ if $E(p^1) > E(p^2)$, and $0 < \sigma < 1$ only if $E(p^1) = E(p^2)$, where $E(p^j)$ is the expected price from the position j, j $\in \{1, 2\}$. Second, anticipating the equilibrium play in the second stage, the firms have no profitable deviation in their bidding strategies in the first stage. Note that now the price competition consists of two levels: One is to compete for sophisticated nonshoppers by price expectation, and the other is to compete for shoppers by realized price.

We can conduct a similar analysis as in the baseline model, though with more complexity (for brief analysis, see Web Appendix B at http://www.marketingpower.com/jmrjune11). Figure 5 illustrates the equilibrium outcome with $\beta = 1/4$. Following the basic patterns found in the baseline model, in the shadowed region in Figure 5, Panel A, the high-type firm does not value the prominent position at all. The high-type firm can win the auction only when either prominence difference or competence difference is significant (the unshadowed region in Figure 5, Panel B), and the low-type firm outbids its rival when both α and c are small (the shadowed region). The expected price from the prominent position is lower when a significant competence difference is

Figure 5
STRATEGIC CHOICE OF ORDERING: β = 1/4

A: Endogenous Valuation

*B: Bidding Outcome*

C: Price Dispersion
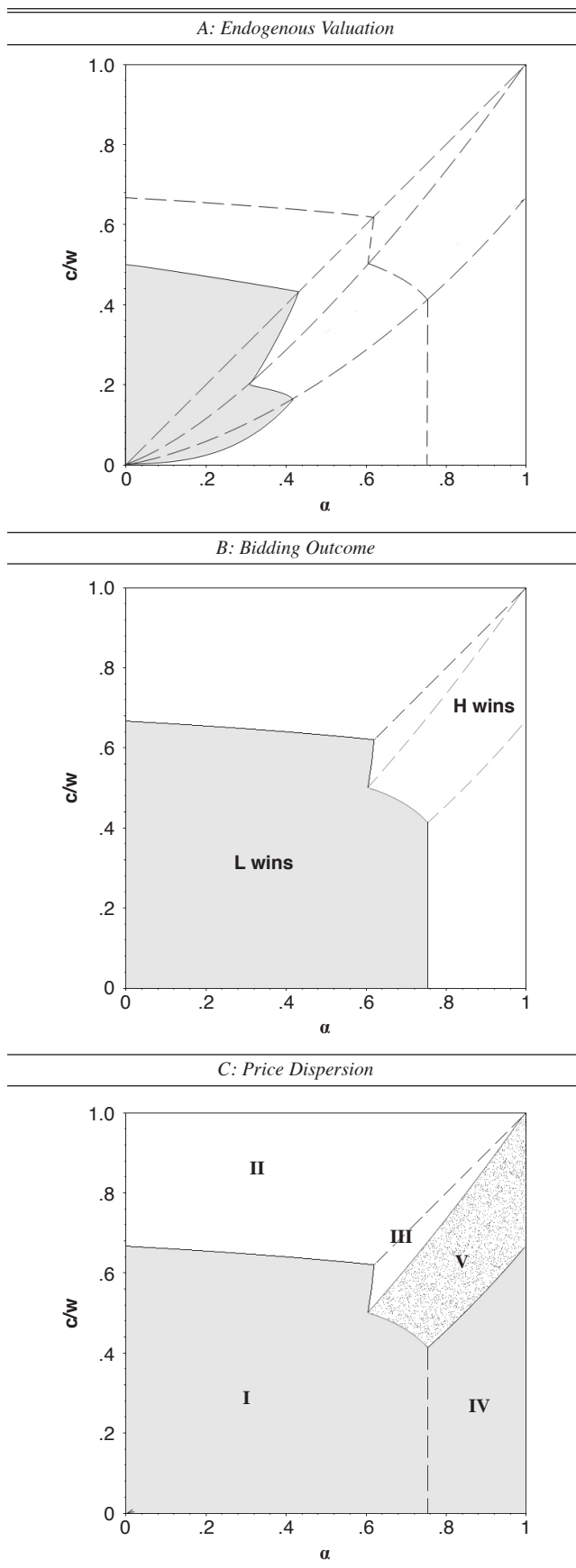
not overwhelmed by the prominence difference (the unshadowed region in Figure 5, Panel C). However, if the cost difference is not so salient, the expected price from the prominent position will be higher (the shadowed region).

Further scrutiny might reveal the effects of consumers' strategic ordering choice on the equilibrium outcomes. Comparing Figure 5, Panel B, with Figure 1, Panel B, the shadowed region expands as a result of the presence of the sophisticated consumers, which implies that the low-type firm has a greater chance of winning the prominent position. This is certainly not because the low-type firm becomes more competitive in this case; instead, it means that the prominent position becomes less attractive to the high-type firm as the portion of sophisticated consumers increases. By staying at the less prominent position and charging a lower price in expectation, the high-type firm not only has a greater chance to win over shoppers but also can attract sophisticated nonshoppers. Expecting the extra demand from the sophisticated consumers, compared with the scenario without sophisticated consumers, the high-type firm is less motivated to acquire the prominent position. This trend is evidenced by the expansion of the "no-interest" region in Figure 5, Panel A (compared with that region in Figure 1, Panel A).

Regarding the expected prices, in the unshadowed region of Figure 5, Panel C, the high-type firm charges a lower expected price and thus attracts all the sophisticated consumers. In the shadowed region, the winning firm charges a higher expected price and thus concedes those sophisticated consumers to its competitor. The winning firm does not charge a lower expected price to attract the sophisticated consumers, because either the winning firm is the low-type firm, which has a cost disadvantage, or the prominence advantage is salient. The dotted region, in which the expected prices from the two positions are the same and thus sophisticated consumers are indifferent in sampling the first or second position, serves as a transition between the two deterministic cases. In this region, the high-type firm wins the prominent position, and the relatively intermediate advantage of the location prominence compared with its cost advantage makes any deterministic choice by sophisticated consumers unsustainable. On the one hand, if all sophisticated consumers begin from the first position for sure, the guaranteed demand becomes too significant to prevent H from exploiting them with a higher price, which contradicts sophisticated consumers' expectation. On the other hand, the relative prominence advantage is not salient enough such that H can afford to forgo all those sophisticated consumers. As a result, H charges the same expected price as L and garners some of the sophisticated consumers, and sophisticated consumers randomize their sampling strategies. With the emergence of such a middle region, in which the expected prices from the two positions are the same, consumers' strategic ordering choice reduces the price dispersion. As the portion of sophisticated consumers increases, firms are more likely to charge the same expected price level.

As the portion of sophisticated consumers continues to grow, the equilibrium pattern slightly changes while the main results of interest remain. Figure 6 illustrates the equilibrium outcomes when β = 3/4. A distinctive feature is the existence of a region in which both firms are indifferent about winning the prominent position (the dotted region in Figure 6, Panels A and B). The reason is as follows:

Figure 6
STRATEGIC CHOICE OF ORDERING: $\beta = 3/4$

A: Endogenous Valuation

B: Bidding Outcome

C: Price Dispersion
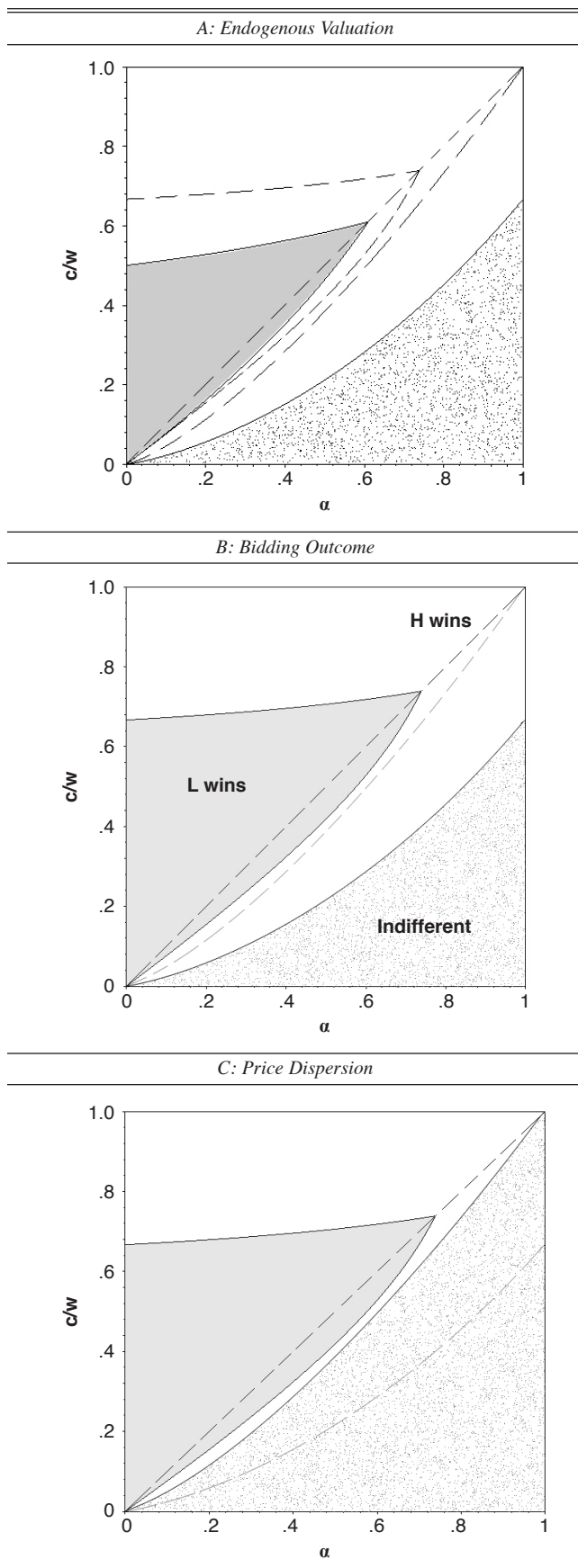
Because the portion of sophisticated consumers is significant enough that their deterministic choice could create a large amount of guaranteed demand, neither type of firm can resist charging a higher price for this guaranteed demand, which in turn contradicts sophisticated consumers' rational expectations. Therefore, no matter which firm wins the prominent position, sophisticated consumers playing pure strategy cannot arise as an equilibrium. Instead, they randomize their sampling in equilibrium. Such mixed strategy gives either firm the same share of guaranteed demand from being in the two positions, which results in firms' indifference between them. In consequence, the price dispersion is further reduced and the expected prices from the two positions are the same in a large region (dotted region in Figure 6, Panel C). When all consumers are sophisticated (i.e., $\beta = 1$), it reduces to an extreme case in which the first position's advantage vanishes completely, and there is essentially no difference between the two positions.

Endogenous Sequential Search

The analysis thus far has treated the consumer sequential search decision as exogenous; that is, some portion of consumers are assumed to search only once, whereas others are assumed to sample both positions. This seemingly strong assumption actually reflects the equilibrium outcome when consumers are allowed to make their sequential search decision endogenously. We now extend the baseline model to endogenize consumers' sequential search decision (i.e., whether to continue or stop searching). As the subsequent discussion indicates, as long as both a commonly observed search ordering and a certain portion of consumers with nonpositive search cost exist, the equilibrium bidding outcome and price dispersion pattern derived from the baseline model continue to hold.

Following the framework in the baseline model, we modify the setup about consumer search behavior as follows. The previous section explained the effect of strategic choice of ordering. Here, we assume that all consumers follow the presumed search ordering to simplify analysis. Suppose that all consumers first sample the prominent position and learn the price, and then they assess the expected gain from an additional search. If the expected gain from the additional search exceeds the search cost, they proceed to sample the second position, compare the prices, and purchase from the position with the lower price; otherwise, they stop searching and purchase from the first position (provided the price does not exceed their willingness to pay w). Following the common assumption in literature (e.g., Stahl 1989), we assume that all consumers sample at least one position. We consider consumers with heterogeneous search costs. Particularly, assume that $1 - \alpha$ of the consumers have zero search cost, and α of them have positive search cost k ($0 < k < w$).[8] As in the preceding case, firms first submit their bids, and then firms and consumers observe the bidding outcome and decide pricing and searching strategies simultaneously.

We first characterize consumers' optimal searching strategies. Given the pricing strategy of the firm in the sec-

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[8]Alternatively, we could allow k to vary following a certain distribution over some positive interval. In that case, the mixed-strategy pricing continues to be the only possible outcome, but in general, there is no closed-form solution (see Stahl 1996).

ond position $F(\cdot)$ (which again is a cumulative distribution function defined on a price support $[\underline{p}, \overline{p}]$), for any individual consumer, the expected gain from sampling the second position, after already knowing the price p from the first position, can be formulated as follows:

$$(5) \qquad G(p) = \int_{\underline{p}}^{p} (p - x)\, dF(x).$$

Note that the price from the first position, p, must belong to the interval $[\underline{p}, \overline{p}]$ because of the common support properties discussed previously. Because $G(p) \geq 0$, it is always optimal for those shoppers with nonpositive search cost to conduct an additional search. However, for those nonshoppers with positive search cost, it is worthwhile to sample the second position only if $G(p) > k$. Similar to Weitzman (1979), we can equivalently define a reserve price r, such that

$$(6) \qquad \int_{\underline{p}}^{r} (r - x)\, dF(x) = k.$$

When the price quoted from the first position, p, exceeds r, it is profitable for nonshoppers to conduct an additional search; otherwise, they will stop searching and purchase from the first position.

After characterizing consumers' optimal searching strategies, we can specify the equilibrium concept. A subgame perfect equilibrium is a strategy profile $\{b_i, F_i(\cdot; s_H, s_L), r(s_H, s_L) : i \in \{H, L\}\}$ such that, first, observing the bidding outcome, $\{F_H(\cdot), F_L(\cdot), r\}$ is an equilibrium in the second stage. In other words, given both firms' pricing strategies $F_H(\cdot)$ and $F_L(\cdot)$, shoppers always sample both positions, whereas nonshoppers sample the first position, learn the price p, and proceed to sample the second position if and only if $p > r$, where r is defined by Equation 6 by substituting the pricing strategy of the firm at the second position for $F(\cdot)$. Meanwhile, given consumers' sequential search strategy (specified by r) and the other firm's pricing strategy $F_{i'}(\cdot)$, neither firm has a profitable deviation. Second, anticipating the equilibrium play in the second stage, neither firm has a profitable deviation in its bidding strategy; that is, $\{b_H, b_L\}$ is an equilibrium in the first-stage bidding competition. Following a similar approach to that of the baseline model, we can derive the equilibrium mixed-strategy pricing and the corresponding expected profits of both firms and then compare their bidding amounts.

The price competition in the second stage follows a similar pattern to that in the baseline model, except firms must take account of nonshoppers' reaction when setting the highest price they can charge. In particular, when H wins the first position, firms' equilibrium pricing strategies share a similar format to the baseline model but with a different price range:

$$(7) \qquad F_H(p) = \begin{cases} \dfrac{p - m}{p - c} & p \in [m, r) \\ 1 & p = r \end{cases}$$

$$F_L(p) = \begin{cases} \dfrac{p - m}{(1 - \alpha)p} & p \in [m, r) \\ 1 & p = r \end{cases},$$

where r is the reserve price for consumers with a positive search cost defined as $r = \min\{(1 - \alpha)k/(1 - \alpha + \alpha \ln\alpha), w\}$, and $m = \max\{\alpha r, c\}$.[9] Similarly, the expected sales profits of both firms in this scenario can be written as $\pi_H^1 = m$ and $\pi_L^2 = (1 - \alpha)(m - c)$.

When L wins the first position, the reserve price of those nonshoppers becomes $r' = \min\{(1 - \alpha)k/(1 - \alpha + \alpha \ln\alpha) + c, w\}$, and the equilibrium pricing strategies can be written as follows:

$$(8) \qquad F_H(p) = \dfrac{p - c - \alpha(r' - c)}{(1 - \alpha)(p - c)} \qquad p \in [c + \alpha(r' - c), r']$$

$$F_L(p) = \begin{cases} \dfrac{p - c - \alpha(r' - c)}{p} & p \in [c + \alpha(r' - c), r') \\ 1 & p = r' \end{cases}.$$

In this case, H achieves an expected profit level of $\pi_H^2 = (1 - \alpha)[c + \alpha(r' - c)]$, and L attains $\pi_L^1 = \alpha(r' - c)$.

We summarize the equilibrium bidding outcome and price dispersion in Figure 7 (in which we arbitrarily set $k = w/5$). As the figure indicates, the equilibrium outcome pattern remains unchanged. The prominent position is always profitable for the low-type firm, and the high-type firm does not value the prominent position sometimes (shadowed region in Figure 7, Panel A). Only when both α and c are small (the shadowed region in Figure 7, Panel B) can the low-type firm win the first position. The expected price from the first position is higher than that from the second one, unless the cost advantage is overwhelming (the unshadowed region in Figure 7, Panel C).

When considering consumers' sequential search strategies, especially when nonshoppers' search costs are not too high, the firm at the prominent position no longer fully exploits them by setting the upper bound of the price support equal to consumers' total surplus w. The reason is simply that the firm cannot afford to lose the entire market. By charging an upper bound price as high as w, the firm would not only lose those shoppers but also invite nonshoppers to conduct an additional search and lose them as well. Realizing this, the firm with location advantage adjusts its price to retain those consumers with relatively high search costs by setting the upper bound of price support equal to nonshoppers' reserve price to keep them from further searching. Correspondingly, the firm at the less prominent position adjusts its price as well to compete for the shoppers who sample both positions. Such automatic adjustment of pricing lowers the equilibrium prices from the two positions simultaneously, with the relative price pattern and the relative equilibrium profits unchanged. As the comparative result, both the equilibrium bidding outcome and the comparison of price expectation exhibit no substantial change in pattern.

The analysis and results on consumer sequential search are also consistent with recent empirical findings. For example, Kim, Albuquerque, and Bronnenberg (2009) estimate a structural model of consumer sequential search using online product search data from Amazon.com and show that

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[9] In the case when $m = c$, the nonshoppers' actual reserve price r'' could be different from r, but $r \leq r'' < c/\alpha$ so that it does not affect the equilibrium outcomes.

Figure 7
ENDOGENOUS SEQUENTIAL SEARCH: k = w/5

A: Endogenous Valuation

B: Bidding Outcome

C: Price Dispersion
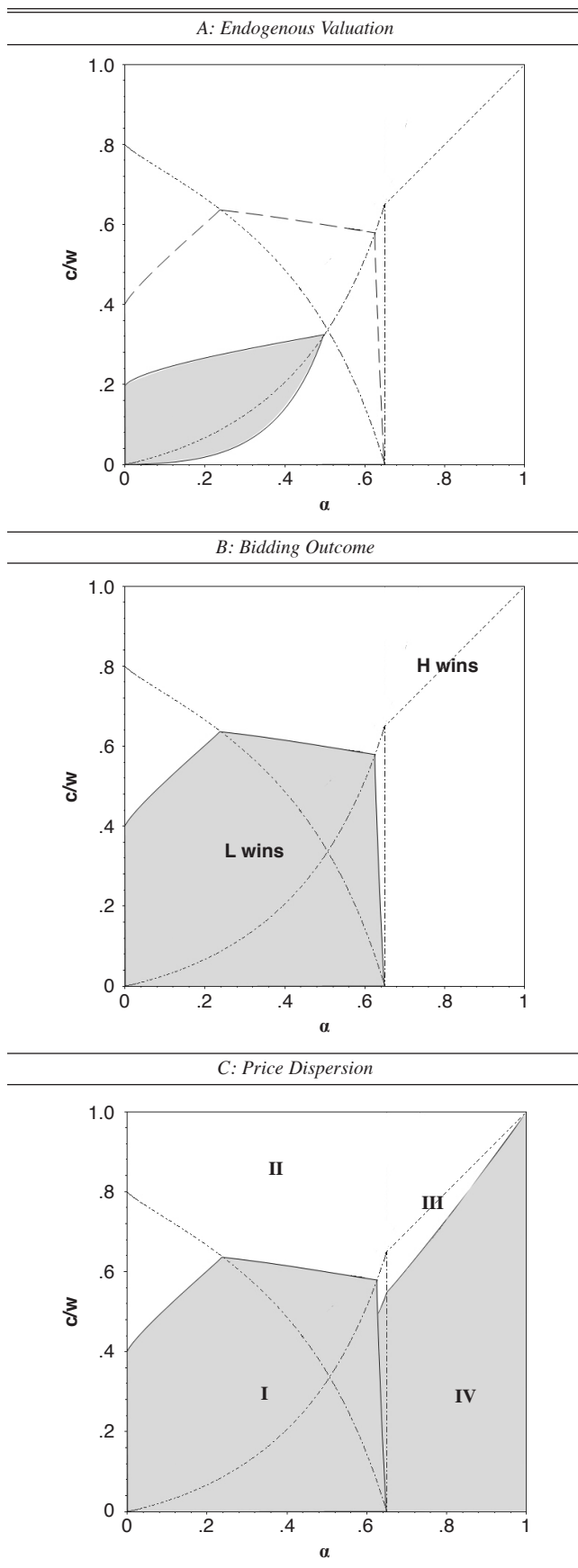
consumers have different search costs and that high-cost consumers perform limited search.

EXTENSION AND DISCUSSION

External Information Channels

In the baseline model, we assume that all consumers obtain the product information from search advertising. Now we relax this assumption and consider the case in which consumers can obtain product information from other channels. In particular, we now assume that among all consumers (with total mass 1), only $1 - M$ ($0 < M < 1$) of them obtain price information from search advertising (i.e., from the two advertising positions considered here). The other M portion of consumers obtain price information from external channels (e.g., newspapers, television) and are assumed to be aware of both firms' product information. Moreover, we can also consider different information coverage rates of firms in the outside channels and further consider overlap of information coverage between different channels, which can be shown to not affect the qualitative results. Firms charge the same price to both the search advertising and outside markets. (If the pricing decisions are made separately, these are essentially separate markets.) In summary, $M + (1 - M)(1 - \alpha)$ of consumers (i.e., consumers from the outside channels plus shoppers in the search market) are informed of both firms' prices and purchase from the one offering a lower price, whereas $(1 - M)\alpha$ of consumers (i.e., nonshoppers in the search market) sample the firm at the first position only and purchase from there (if the price does not exceed w).

We can derive firms' pricing strategies accordingly. For example, when H wins the first position, the equilibrium pricing is as follows:

$$(9) \qquad F_H(p) = \begin{cases} \dfrac{p - \underline{p}}{p - c} & p \in [\underline{p}, w) \\ 1 & p = w \end{cases}$$

$$F_L(p) = \begin{cases} \dfrac{p - \underline{p}}{[(1 - M)(1 - \alpha) + M]p} & p \in [\underline{p}, w) \\ 1 & p = w \end{cases},$$

where $\underline{p} = \max\{(1 - M)\alpha w, c\}$. When H wins, firms' expected sales profits are $\pi_H^1 = \underline{p}$ and $\pi_L^2 = [(1 - M)(1 - \alpha) + M](\underline{p} - c)$; when L wins, firms achieve profit levels $\pi_L^1 = (1 - M)\alpha(w - c)$ and $\pi_H^2 = [(1 - M)(1 - \alpha) + M][(1 - M)\alpha(w - c) + c]$. Firms bid $b_i = \max\{\pi_i^1 - \pi_i^2, 0\}/(1 - M)$ per click and are ranked based on the score $s_i = \max\{\pi_i^1 - \pi_i^2, 0\}$, where $i \in \{H, L\}$. Note that when $M = 0$, all results reduce to those from the baseline model.

Figure 8 illustrates the results when $M = 1/3$. The high-type firm's endogenous valuation, the bidding outcome, and the spatial price dispersion all follow patterns similar to those in the baseline model. For example, H achieves higher profit in the second position than the first one in the shadowed region in Figure 8, Panel A, and the expected price from the first position is lower in the unshadowed region in Figure 8, Panel C.

An aspect worth noting is that as M increases, the regions in which H does not value the first position and in which H fails to outbid L both expand rather than shrink (compared

Figure 8
EXTERNAL INFORMATION CHANNELS: M = 1/3

A: Endogenous Valuation

B: Bidding Outcome

C: Price Dispersion
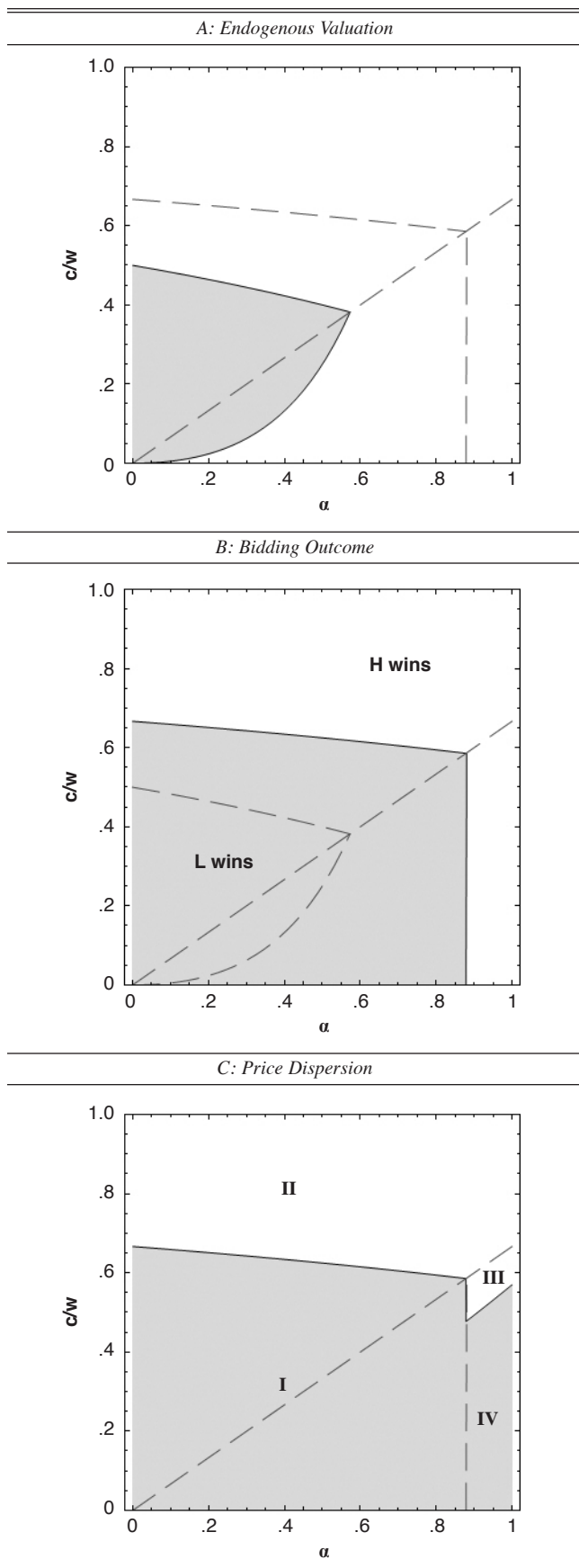
with the baseline model in which M = 0). It implies that when the search advertising market is only part of the entire product market, as it is in reality, the trade-off indicated in this study is even more salient. Recall that two counterbalancing effects determine the profitability of winning the prominent position for the high-type advertiser: capturing nonshoppers when winning the position versus benefiting from a higher premium charged to shoppers when letting the weaker competitor take the location advantage. When M increases, the relative size of the search market decreases and thus the loss from losing the nonshoppers decreases, which reduces the relative significance of the first effect. Meanwhile, the extra demand from the external market increases the benefit from weakening the price competition and raising the equilibrium prices, which enforces the second effect. As a result, the first position actually becomes less appealing to the high-type firm.

Heterogeneous Consumer Preference

Next, we relax the homogeneous product assumption and allow consumers to have heterogeneous preferences. Starting from the baseline model, similar to Narasimhan (1988), we now assume that among all consumers (with total mass 1), t_1 of them are loyal to H's product, t_2 of them are loyal to L's product, and the rest $1 - t_1 - t_2$ ($0 < t_1 + t_2 < 1$) do not have a particular preference and purchase from the firm offering a lower price. Assume firm i's loyal customers visit firm i's position directly and buy if the price does not exceed w ($i \in \{H, L\}$). The rest of the consumers follow the same search pattern: α of them are nonshoppers and $1 - \alpha$ are shoppers.

Following a similar analysis, we can derive the equilibrium outcome. For example, when H wins, the pricing strategies are as follows:

$$(10) \quad F_H(p) = \begin{cases} \dfrac{[(1 - t_1 - t_2)(1 - \alpha) + t_2](p - \underline{p})}{(1 - t_1 - t_2)(1 - \alpha)(p - c)} & p \in [\underline{p}, w) \\ 1 & p = w \end{cases}$$

$$F_L(p) = \begin{cases} \dfrac{(1 - t_2)(p - \underline{p})}{(1 - t_1 - t_2)(1 - \alpha)p} & p \in [\underline{p}, w) \\ 1 & p = w \end{cases},$$

where $\underline{p} = \max\{[(1 - t_1 - t_2)\alpha + t_1]w/(1 - t_2), (w - c)t_2/[(1 - t_1 - t_2)(1 - \alpha) + t_2] + c\}$, and firms' expected profits are $\pi_H^1 = (1 - t_2)\underline{p}$ and $\pi_L^2 = [(1 - t_1 - t_2)(1 - \alpha) + t_2](\underline{p} - c)$. The per-click bids are $b_H = \max\{\pi_H^1 - \pi_H^2, 0\}/(1 - t_2)$ and $b_L = \max\{\pi_L^1 - \pi_L^2, 0\}/(1 - t_1)$, whereas the scores are still $s_i = \max\{\pi_i^1 - \pi_i^2, 0\}$ ($i \in \{H, L\}$), as indicated previously. Figure 9 illustrates the results when $t_1 = t_2 = .1$, showing that the pattern remains unchanged.

Consumers' heterogeneous preferences affect the results only to the extent that they change the total size of the market for which firms compete. Nevertheless, as long as there is still a certain portion of consumers willing to switch between products, the aforementioned trade-off remains. Therefore, the results of interest change only quantitatively rather than qualitatively.

Figure 9
HETEROGENEOUS CONSUMER PREFERENCE: $t_1 = t_2 = .1$

A: Endogenous Valuation

B: Bidding Outcome

C: Price Dispersion
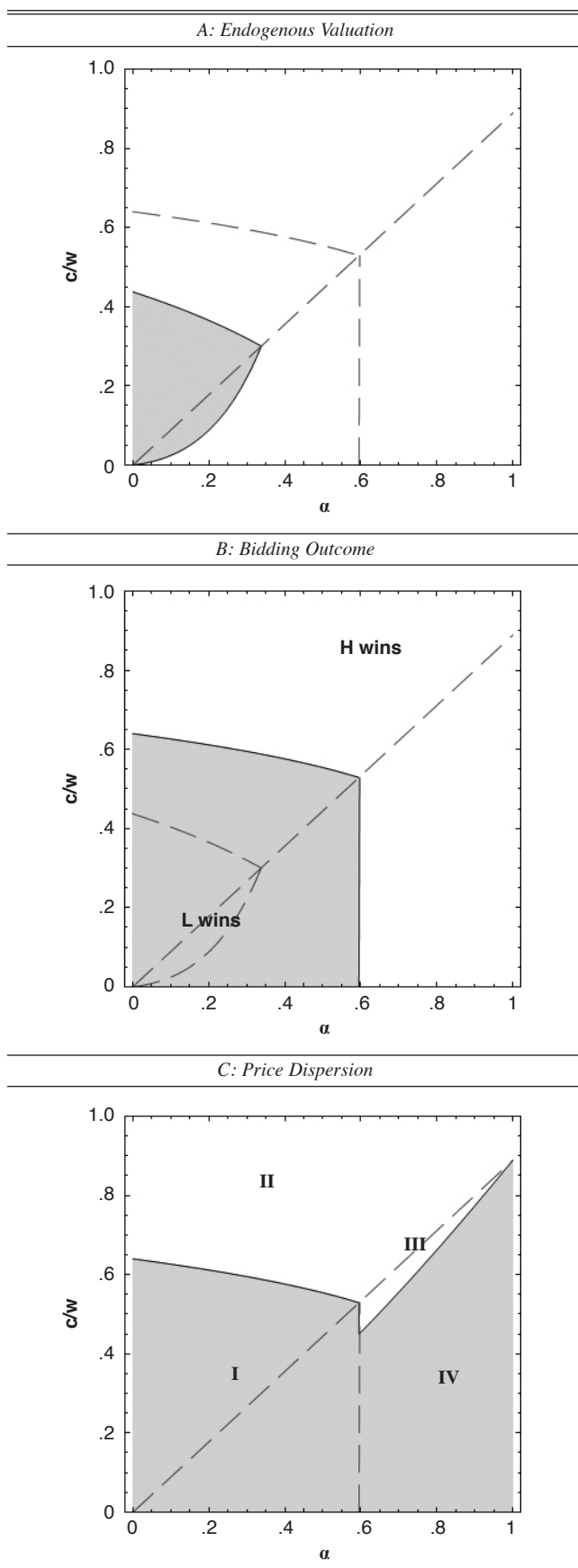
Multiple Competing Firms

Although mainly based on duopoly analysis, the results hold beyond the case of only two firms. In this section, we consider a case of three competing firms to show that the main results can be extended to the oligopolistic setting.

Extending the baseline model, we now consider three firms with different production costs c_1, c_2, and c_3. We consider a simple case in which $c_1 = c_2 > c_3$. (The more general case that $c_1 \geq c_2 \geq c_3$ would add further technical discussion, and the qualitative results of interest can be expected to hold.) Without loss of generality, we normalize $c_3 = 0$ and denote $c_1 = c_2 = c$ ($0 < c < w$). Again, we call the low-cost firm the high type, or H, and the two high-cost firms the low type, or L. There are three advertising positions with different prominence levels, reflecting consumers' search ordering. Similarly, assume that all consumers (with total mass 1) begin searching from the first position; among them, α_1 stop and purchase from the first position if the price does not exceed w. The rest, $1 - \alpha_1$, continue searching and sample the second position; α_2 of them (i.e., with a total mass $\alpha_2[1 - \alpha_1]$) stop searching after knowing the first two firms' prices and buy from the one offering the lower price. The rest $(1 - \alpha_1)(1 - \alpha_2)$ are shoppers, who sample all three firms and buy from the one offering the lowest price. For ease of exposition, we let $\alpha_1 = \alpha_2 = \alpha$ ($0 < \alpha < 1$). The analysis and results can be naturally extended if α_1 and α_2 take different values. The other settings follow the baseline model.

The price competition among three firms becomes much more complex than the duopoly case. Web Appendix C (http://www.marketingpower.com/jmrjune11) details a complete description of the equilibrium pricing. Figure 10 illustrates two equilibrium price patterns that exhibit noteworthy features. Figure 10, Panel A, depicts the cumulative distribution functions for the equilibrium pricing when H stays in the first position and $c/w < \alpha^2/[1 + \alpha(1 - \alpha)]$. Note the stair shape of the equilibrium price supports. Overlap of price supports exists only between directly adjacent firms. There is no direct price competition between firms placed far from each other. The localized competition reflects the subtle interaction between cost advantage and location advantage. With limited cost advantage (i.e., c/w is relatively small), H would rather take advantage of its good location by charging high prices than enter the competition for shoppers with very low prices. Likewise, getting into the high price range is not profitable for L in the third position, which has neither cost nor location advantage. In contrast, Figure 10, Panel B, represents the case in which H stays in the first position and $\alpha^2/[1 + \alpha(1 - \alpha)] < c/w < \min\{\alpha,[1 - \alpha(1 - \alpha)]/[1 + \alpha(1 - \alpha)]^2\}$. The probability mass near the lower bound of H's price support (which does not appear in Figure 10, Panel A) indicates that with considerable cost advantage, H is willing to compete for shoppers with more competitive prices. This unique pricing pattern that involves segmented price supports and localized price competition is absent in the typical price competition literature.[10]

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[10]A few studies analyze similar local-competition pricing patterns as in Figure 10, Panel A (e.g., Xu, Chen, and Whinston 2011) or segmented price supports (e.g., Kannan 2010), which are different from the unique pattern illustrated in Figure 10, Panel B.

Figure 10
EQUILIBRIUM PRICING WITH THREE FIRMS

Figure 11
ENDOGENOUS VALUATION IN THE CASE OF THREE FIRMS

nously investigate the price competition, the weaker firms tend to charge less competitive prices when placed in the good positions, which leaves a higher profit margin for the stronger firm in a lower position and thus reduces its bidding incentive. Therefore, the trade-off of interest remains, and similar results can be derived.

Supportive Observations

Although the new perspective on location choice and pricing decisions we proposed in this study could easily be neglected by some marketing managers in practice, there are many empirical observations consistent with the results from our modeling analysis. We provide some examples in this section.

To investigate advertisers' bidding behaviors in reality, we track the actual sponsored ranking results from the leading online search engine, Google. A program was designed to automatically enter search queries using the given keywords every five minutes and to record the ranks of targeted firms' sponsored links for three consecutive weeks beginning at the noon on May 18, 2010. Note that in addition to the regular sponsored links on the right side of the web page, Google also provides premium sponsored positions in a highlighted region right above the general search results, which are much more noticeable and usually much more costly. Thus, we rank the premium sponsored positions higher than the regular ones. For example, if there are two premium positions, the first regular sponsored link is ranked number three. We chose keywords to fit our model setting as closely as possible. Table 3 summarizes the statistics of the data recorded.

The observations shown in Table 3 can be well interpreted by our model results. Both textbooks and photo prints are a relatively standard product or service, so price would be the primary consideration. Textbooks.com is a
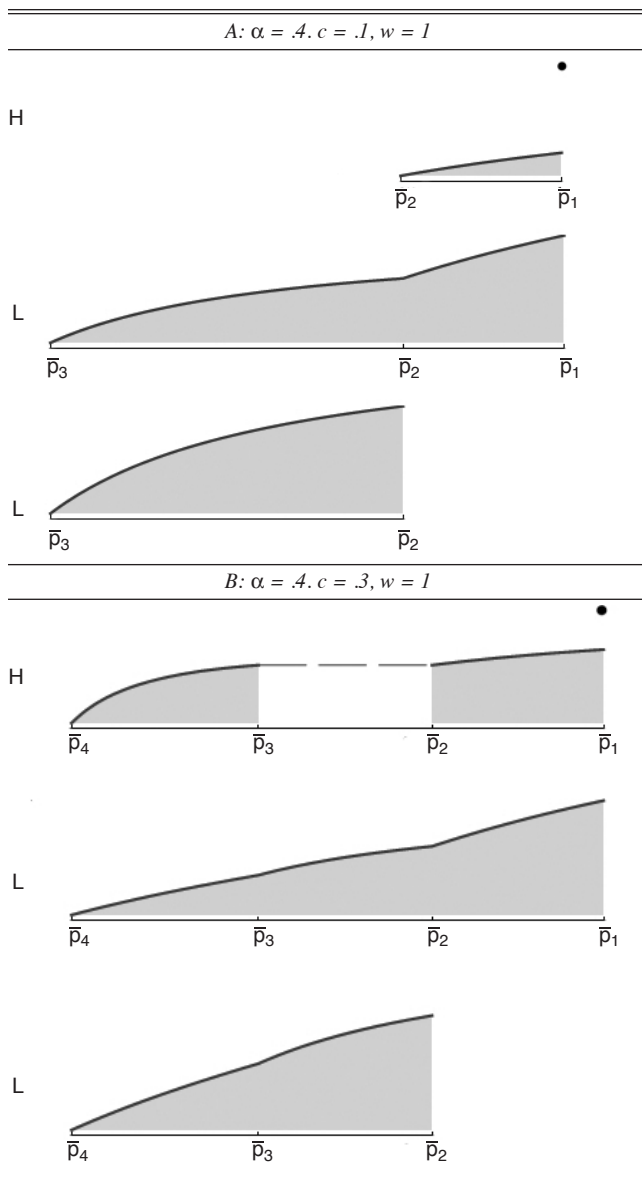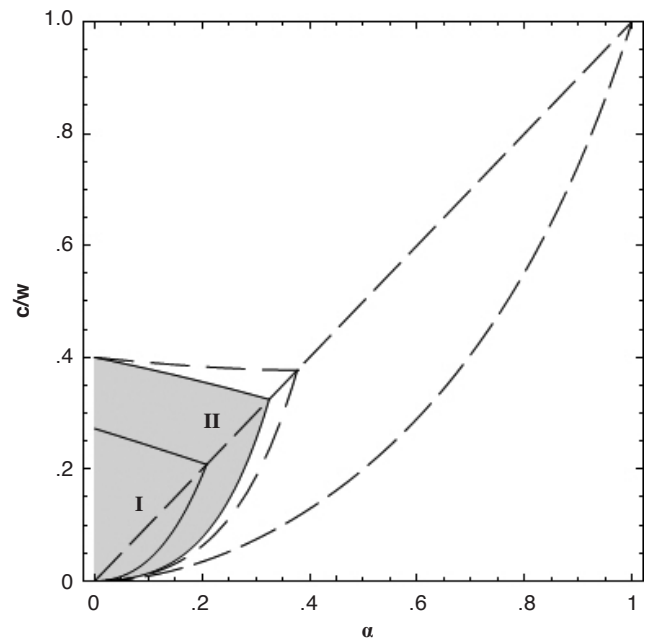
Table 2 summarizes firms' equilibrium profits from price competition under different scenarios. If we compare H's profits in different positions, similar results arise in the three-firm case: When evaluating endogenously in the product market competition, a less prominent position might not mean less profit. As Figure 11 shows, in Region I, the third position generates the highest equilibrium profit for H among all three positions (i.e., $\pi_H^3 > \pi_H^1 > \pi_H^2$); in Region II, the third position is more profitable than the second one (i.e., $\pi_H^1 > \pi_H^3 > \pi_H^2$). In other words, in the shadowed region, the "worst" position actually outperforms a "better" position for the high-type firm. In addition to the endogenous valuation, similar results on the equilibrium bidding outcome and spatial price dispersion pattern can be derived as well (for details, see Web Appendix D at http://www.marketingpower.com/jmrjune11).

In the case of multiple firms with different competitive strength competing against each other, when we endoge-

Table 2

EQUILIBRIUM PROFITS FROM PRICE COMPETITION IN THE CASE OF THREE FIRMS

	H-L-L	L-H-L	L-L-H
When $c/w < \alpha^2/[1 + \alpha(1 - \alpha)]$			
Profit in first position	αw	$\alpha(w - c)$	$\alpha(w - c)$
Profit in second position	$\alpha(1 - \alpha)\{w/[1 + \alpha(1 - \alpha)] - c\}$	$\alpha(1 - \alpha)[w + (1 - \alpha)c]/[1 + \alpha(1 - \alpha)]$	$\alpha(1 - \alpha)(\overline{p}_2 - c)$
Profit in third position	$\alpha(1 - \alpha)^2\{w/[1 + \alpha(1 - \alpha)] - c\}$	$(1 - \alpha)^2\{(\alpha w - c)/[1 + \alpha(1 - \alpha)]\}$	$(1 - \alpha)^2\overline{p}_3$
When $\alpha^2/[1 + \alpha(1 - \alpha)] < c/w < \alpha$			
Profit in first position	αw	$\alpha(w - c)$	$\alpha(w - c)$
Profit in second position	$(1 - \alpha)(\alpha w - c)$	$\alpha(1 - \alpha)[w + (1 - \alpha)c]/[1 + \alpha(1 - \alpha)]$	$\alpha(1 - \alpha)(\overline{p}_2 - c)$
Profit in third position	$(1 - \alpha)^2(\alpha w - c)$	$(1 - \alpha)^2\{(\alpha w - c)/[1 + \alpha(1 - \alpha)]\}$	$(1 - \alpha)^2\overline{p}_3$
When $c/w > \alpha$			
Profit in first position	c	$\alpha(w - c)$	$\alpha(w - c)$
Profit in second position	0	$(1 - \alpha)c$	$\alpha(1 - \alpha)(\overline{p}_2 - c)$
Profit in third position	0	0	$(1 - \alpha)^2\overline{p}_3$

Notes: 1. H-L-L represents the case in which H stays in the first position, and similar for the others.

2.
$$\overline{p}_2 = \frac{-(1 - \alpha)(1 - 2\alpha)c + w + \sqrt{5(1 - \alpha)^2c^2 - 2(1 - \alpha)(1 - 2\alpha)wc + w^2}}{2[1 + \alpha(1 - \alpha)]}.$$

3. $\overline{p}_3 = \alpha(\overline{p}_2 - c) + c.$

Table 3

SUMMARY STATISTICS OF THE SPONSORED RANKING DATA (TOTAL TIME PERIODS: 6048)

	Appearance	*Mean Rank*	*SD*
Keyword: Textbooks			
Textbooks.com	5993	1.065076	.260501
Amazon.com	6040	2.443543	.743103
Keyword: Online Photo Print			
YorkPhoto.com	6042	1.002979	.070407
Shutterfly.com	6040	2.395530	.717865
Keyword: Car Rental			
Budget	6043	2.098957	.922594
Avis	6040	2.488079	1.530998
Enterprise	5466	4.600256	2.893965
Keyword: Dell Laptop			
Dell	6042	1.000331	.025730
Staples	5715	4.280315	1.220876

website selling new and used textbooks. In contrast, Amazon.com, as the largest online bookstore and marketplace, could have lower average marginal costs, probably owing to economy of scale, better managed information systems, or greater bargaining power over the supply chain. Thus, we might consider Amazon.com as the high-type firm. Similarly, compared with the NASDAQ-listed leading digital photo service company, Shutterfly, YorkPhoto.com is smaller in scale and probably weaker in competitive strength. Nevertheless, given the highly standardized products, it can be expected that the cost differences should be small in both cases. Because the price-quoting process is relatively straightforward in both cases, consumers can easily compare prices, which could result in a small α value. As is shown, when α and c are small, the low-type firm may have higher bidding incentive, which is reflected by the consistently higher ranks of both Textbooks.com and YorkPhoto.com.

In the car rental example, North America's largest rental car company, Enterprise, stays at a lower sponsored rank in general, which can be interpreted similarly to the previous two examples. The high variance in its ranking is worth attention. Because the search process is more complex than the previous two examples, which potentially corresponds to a higher α value, and because the cost advantage of the market leader versus smaller firms might also increase, according to our results, with an increased α and/or c, the high-type firm's bidding incentive might increase accordingly. From the collected data, Enterprise wins the first position about one-fourth of the time and stays in the fifth or lower positions more than half the time. It fits our results in the case of multiple competing firms: With considerable α and/or c values, H may adopt mixed-strategy bidding and either outbid both L firms or stay in the lowest position (for details, see Web Appendix D at http://www.marketingpower.com/jmrjune11).

The fourth example corresponds to the case of dominating cost advantage. Because Google restricts the bidding to brand names, only authorized firms can bid for keywords containing particular brand names. Staples is Dell's authorized retailer and also is the competing channel of Dell's direct sales. When they compete in search advertising, their decisions may be considered roughly independent. Producing and selling the same laptops, Dell undoubtedly possesses significant cost advantage. Similar to the model results, Dell tightly holds the best advertising position, with no exceptions.

There is also evidence supporting the results on equilibrium pricing. In addition to the aforementioned literature, websites that observe the real-time product prices on Amazon.com find significant levels of temporal price fluctuation in various product categories. Figure 12 shows some findings from one such website. The spatially differentiated price expectation pattern can also be examined using the examples given. For example, in general, Amazon.com offers more competitive textbook prices, and Dell online store is believed to sell cheaper Dell laptops than other retailers,[11] consistent with the model predictions.

In addition, other model results are also supported by real-world data. For example, recall that, as Figure 4 shows, the bidding competition is the keenest when α is large and c is small. A website called CyberWyre (www.cwire.org) keeps an updated list of the highest paying search terms. Currently, the most expensive search terms are mesothelioma-related lawsuits, which can cost as high as $69.1 per click. As reported in a *New York Times* article (Liptak 2007), mesothelioma cases are relatively routine and "settle rather

easily," which indicates negligible cost differences. In contrast, the search process can be time-consuming because lawyers "will steer [people] into highly tendentious information" to capture these clients, which results in a high α value. As a result, law firms "compete on Google instead of competing on price," which fits the model results that firms bid aggressively and charge a high price upon winning under such circumstances.
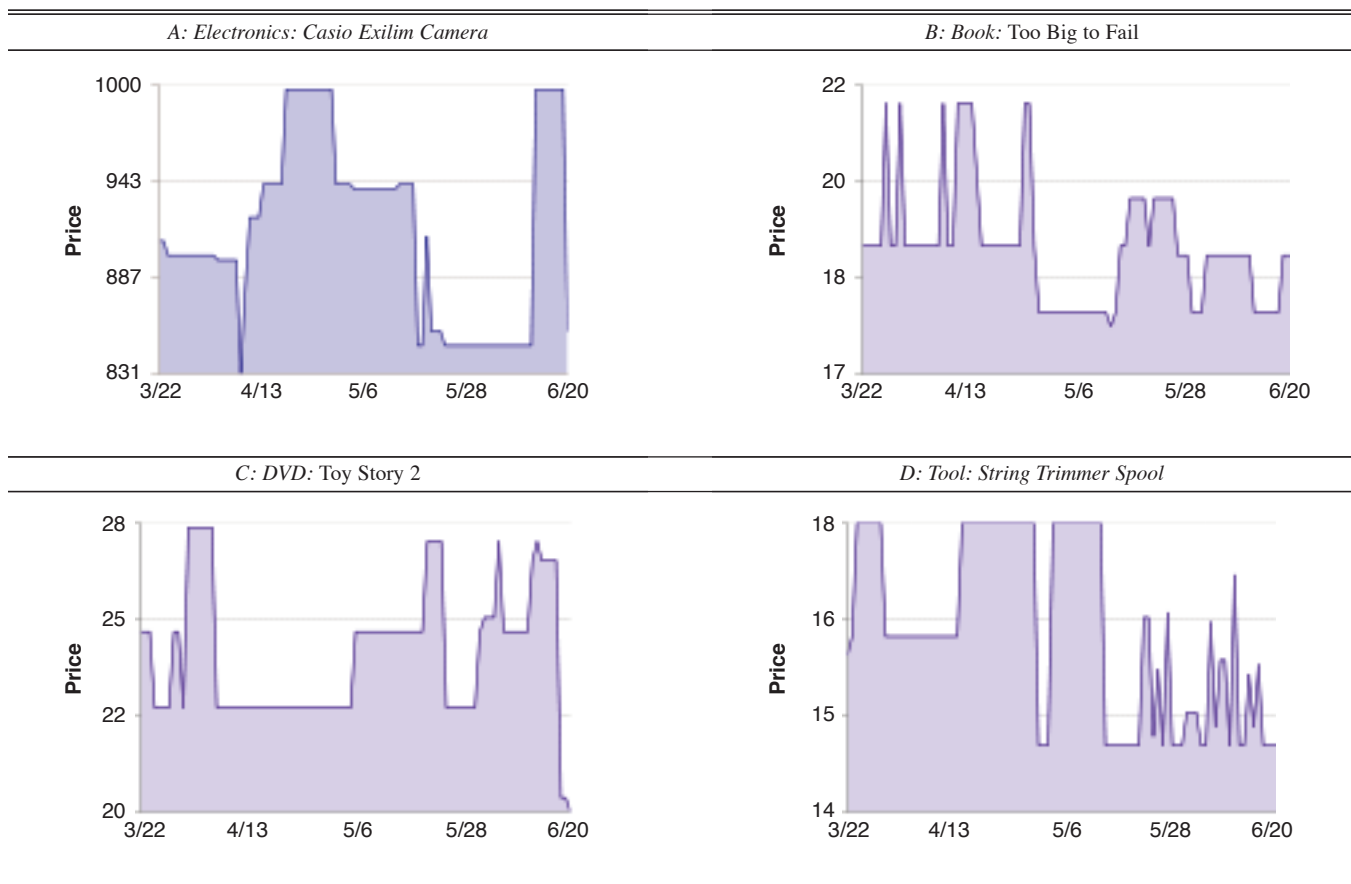
CONCLUSION

When marketing managers deal with location choice, such as competing for online advertising slots, understanding the value of a premium location is a fundamental issue. Only if they comprehend the value difference between locations can managers optimally allocate their spending to achieve the best possible marketing results. In this study, we investigate the value of a prominent advertising position endogenously in the context of price competition among asymmetric advertisers in the search advertising setting. We examine the equilibrium outcome of the bidding competition, as well as the resulting price dispersion pattern in various scenarios.

Compared with the existing literature, we illustrate that, in search advertising, the value of the advertising slots should be determined endogenously in price competition rather than taken for granted exogenously. For a particular advertiser, the per-click value, instead of being fixed, could

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[11]A random price comparison on June 9, 2010, indicates that the classical microeconomics textbook *Microeconomic Theory* is sold for $114.94 on Amazon.com but $118.68 on Textbooks.com. Another random price check on June 27, 2010, indicates that the Dell Inspiron 15-inch laptop is sold for $639.98 on Staples.com but it can be bought from Dell.com for $584.99 with the same configuration.

Figure 12
DAILY PRICES OF DIFFERENT PRODUCTS ON AMAZON.COM

A: Electronics: Casio Exilim Camera

B: Book: Too Big to Fail

C: DVD: Toy Story 2

D: Tool: String Trimmer Spool

Source: http://www.frozenwarrior.com/~pricewatch

vary across slots depending on the competitor it faces and how consumers search. A prominent advertising slot is not always desirable, even if it is cost free. We identify a sophisticated pattern of price dispersion resulting from the unique features of online consumer search behavior. This study is among the few focusing on the asymmetric competition among advertisers on the issue of price advertising.

Our analysis has several implications for marketing managers. We underscore the fact that advertisers' willingness to pay for prominent locations should not be determined in isolation. In-depth investigation of a firm's relative competitive strength in the industry is crucial to determine firms' advertising spending. Firms in different competitive situations should tailor their advertising strategies accordingly. In particular, a firm with competitive advantage in some cases could even be better off by staying at a less prominent position and by pricing properly to soften price competition. Such competence-dependent evaluation calls for coordination and communication between marketing teams and other business functions (e.g., production, sales) in a company.

Likewise, thorough market investigation regarding consumer search behavior is indispensable. Consumer search patterns may vary across products (e.g., books vs. computers) or across time periods (e.g., weekdays vs. weekends). Thanks to the advances of information technology, the investigation can be conducted at a lower cost and the information collected more easily than ever before (e.g., search engines usually track the clicks of sponsored links at different ranks for different keywords).

Most important, our analysis provides the rationale for firms to determine their spending as they compete for a prominent advertising position. The rule of thumb is that both the relative competitive strength and prominence difference matter in determining firms' bidding strategies. Firms that have a competitive advantage, when neither their competitive advantage nor the location prominence difference is salient, should forgo the most prominent slots and leverage their revenue instead by lowering the price to capture consumers. In contrast, disadvantaged firms should bid aggressively in this scenario to reap the benefit of the prominent position. When either the competence difference or the prominence difference is significant, disadvantaged firms should avoid being too ambitious and overinvesting in the bidding competition.

The price dispersion patterns derived from our model are also of interest to consumers. Because of the two-dimensional price dispersion, in general, there is no straightforward way to find the lowest price in a one-shot search. For consumers who have low search costs, we recommend conducting a thorough search. For those who are not willing to search extensively, sampling the firms that have competitive advantage (e.g., big branded retailers) might be wise because they are more likely to charge lower prices in general.

Although we use online search advertising as the setting for our discussion, our model and analysis might also apply to other settings involving location acquisition and price competition. This is because the rank of an advertising slot in the online world is similar to the degree of prominence of business locations in the physical world—from stores in a shopping mall, to gas stations along a highway, to shelf space in grocery stores. Consider slotting in supermarkets

as an example. It is commonly believed that product location on the shelf has an important effect on sales, and a central location at eye level is most desirable (Dreze, Hoch, and Purk 1994). Given the significant difference in prominence and the scarcity of prominent shelf space relative to the number of products, firms compete intensely for shelf positions by paying various forms of "slotting allowances," which are lump-sum advance payments made to retailers by manufacturers for stocking their products on the shelf. The results and insight delivered in this article also shed light on slotting allowances, in that manufacturers compete for prominent shelf positions and battle for consumers through pricing, resembling the search advertising case in many ways.

This article triggers directions for further research. Here, firms' valuation of the prominent position is endogenized in the pricing competition; this can be viewed as an example of an unexplored class of auctions in which an object's value to a particular bidder depends on its competitors. The study of such auctions becomes even more noteworthy if extended to a general case in which heterogeneous firms compete for multiple display positions, combining multiple-object auctions and asymmetric oligopoly price competition together.

APPENDIX: PROOFS

Proof of Lemma 1

a. If $\alpha w > c$ and such that $m = \alpha w$, $F_H(p) = (p - \alpha w)/(p - c)$ with a mass point at the upper bound w (with probability $(\alpha w - c)/(w - c)$), and $F_L(p) = (p - \alpha w)/[(1 - \alpha)p]$. We can verify that firms have no profitable deviation. This is because, according to firms' payoff function (denote $\pi_i^j(p)$ as the expected profit of firm i in position j charging price p),

(A1) $$\pi_H^1(p) = \alpha p + (1 - \alpha)p[1 - F_L(p)]$$
$$\pi_L^2(p) = (1 - \alpha)(p - c)[1 - F_H(p)],$$

both firms achieve a constant expected profit level within the support (i.e., $\pi_H^1(p) \equiv \alpha w$, $\pi_L^2(p) \equiv (1 - \alpha)(\alpha w - c)$, $\forall p \in [\alpha w, w)$).

If $\alpha w \leq c$ and such that $m = c$, H takes advantage of its low cost, charging $p_H = c$ for sure ($F_H(p) \equiv 1$, $\forall p \in [c, w]$), which gives L zero profit. Meanwhile, L plays mixed strategy $F_L(p) = (p - c)/[(1 - \alpha)p]$ such that H has no profitable deviation, because $\pi_H^1(p) \equiv c$, $\forall p \in [c, w)$, according to Equation A1.

In addition, in both cases, neither firm has incentive to charge higher than w or lower than m, because the former leads to no purchase and zero profit and the latter is dominated by charging m.

b. Similarly, we can verify that both firms have constant profit level within the support:

(A2) $$\pi_H^2(p) = (1 - \alpha)p[1 - F_L(p)] \equiv (1 - \alpha)[\alpha(w - c) + c]$$
$$\pi_L^1(p) = \alpha(p - c) + (1 - \alpha)(p - c)[1 - F_H(p)] \equiv \alpha(w - c).$$

Again, we can check that there is no profitable deviation by charging outside the given support.

Proof of P_1

According to Table 1,

(A3) $$\Delta\pi_L = \pi_L^1 - \pi_L^2 = \alpha(w - c) - (1 - \alpha)(m - c), \text{ and}$$

(A4) $\Delta\pi_H = \pi_H^1 - \pi_H^2 = m - (1 - \alpha)[c + \alpha(w - c)]$.

For firm L, if $\alpha w \leq c$, $\Delta\pi_L = \alpha(w - c) > 0$; if $\alpha w > c$, $\Delta\pi_L = \alpha(\alpha w - c) + (1 - \alpha)c > 0$. Therefore, $\Delta\pi_L > 0$.

For firm H, if $\alpha w \leq c$, $\Delta\pi_H = \alpha(2 - \alpha)c - \alpha(1 - \alpha)w$, and thus, $\Delta\pi_H < 0$ if and only if $c < (1 - \alpha)w/(2 - \alpha)$. If $\alpha w > c$, $\Delta\pi_H = \alpha^2 w - (1 - \alpha)^2 c$, and thus, $\Delta\pi_H < 0$ if and only if $c > \alpha^2 w/(1 - \alpha)^2$. Note that at $\alpha = (3 - \sqrt{5})/2$, $(1 - \alpha)w/(2 - \alpha) = \alpha^2 w/(1 - \alpha)^2 = \alpha w$ (three lines intersect at one point). Therefore, $\Delta\pi_H < 0$ if and only if $\alpha^2 w/(1 - \alpha)^2 < c < (1 - \alpha)w/(2 - \alpha)$.

Proof of P_2

By Equations A3 and A4, if $\alpha w \leq c$, $\Delta\pi_H - \Delta\pi_L = \alpha[(3 - \alpha)c - (2 - \alpha)w]$, and thus, $\Delta\pi_H > \Delta\pi_L$ if and only if $c > (2 - \alpha)w/(3 - \alpha)$. If $\alpha w > c$, $\Delta\pi_H - \Delta\pi_L = (-\alpha^2 + 4\alpha - 2)c$, and thus $\Delta\pi_H > \Delta\pi_L$ if and only if $2 - \sqrt{2} < \alpha$ (<1). Note that the line $(2 - \alpha)w/(3 - \alpha)$ intersects with line αw at $\alpha = 2 - \sqrt{2}$. Therefore, $\Delta\pi_H > \Delta\pi_L$ if $\alpha > 2 - \sqrt{2}$ or $c > (2 - \alpha)w/(3 - \alpha)$. By P_1, $\Delta\pi_L > 0$ and $\Delta\pi_H < 0$ only when $\alpha^2 w/(1 - \alpha)^2 < c < (1 - \alpha)w/(2 - \alpha)$. So, when $\alpha > 2 - \sqrt{2}$ or $c > (2 - \alpha)w/(3 - \alpha)$, $b_H = \Delta\pi_H > \Delta\pi_L = b_L > 0$. When $\alpha < 2 - \sqrt{2}$ and $c < (2 - \alpha)w/(3 - \alpha)$, $\Delta\pi_H < \Delta\pi_L$, and thus $b_H = \max\{0, \Delta\pi_H\} < \Delta\pi_L = b_L$.

Proof of P_3

a. In this case, L wins the first position. According to Equation 2, $F_L(p) < F_H(p)$, $\forall p \in (c + \alpha(w - c), w]$, which means that the price by firm L first-order stochastically dominates the price by firm H. Thus, $E(p_L) > E(p_H)$.
b. In this case, H wins the first position. If $\alpha w \leq c$, by Equation 1, H plays pure strategy at c while L mixes over $[c,w]$. We conclude that $E(p_H) < E(p_L)$.

If $\alpha w > c$, according to Equation 1,

(A5) $E(p_H) - E(p_L) = \displaystyle\int_{\alpha w}^{w} F_L(p) - F_H(p)\,dp$

$= w\left[(\alpha - c/w)\ln\dfrac{1 - c/w}{\alpha - c/w} + \alpha + \dfrac{\alpha}{1 - \alpha}\ln\alpha\right]$.

Define the preceding equation as $f(\alpha, c, w)$, and note that $\partial f/\partial\alpha = w[\ln(1 - c/w) - \ln(\alpha - c/w) + \ln\alpha/(1 - \alpha)^2 + 1/(1 - \alpha)]$ and $\partial^2 f/\partial\alpha^2 = w\{-1/(\alpha - c/w) + 1/[\alpha(1 - \alpha)^2] + 2\ln\alpha/(1 - \alpha)^3 + 1/(1 - \alpha)^2\}$. Apply Taylor expansion on $\ln\alpha$ at $\alpha = 1$,

(A6) $\ln\alpha = \ln1 - (1 - \alpha) - \frac{1}{2}(1 - \alpha)^2 - ... < \alpha - 1 - \frac{1}{2}(\alpha - 1)^2$.

Therefore,

(A6) $\dfrac{\partial^2 f}{\partial\alpha^2} < w\left[-\dfrac{1}{\alpha - c/w} + \dfrac{1}{\alpha(1 - \alpha)^2}\right.$

$\left. + \dfrac{2[\alpha - 1 - \frac{1}{2}(\alpha - 1)^2]}{(1 - \alpha)^3} + \dfrac{1}{(1 - \alpha)^2}\right]$

$= w\left[-\dfrac{1}{\alpha - c/w} + \dfrac{1}{\alpha}\right] < 0, \forall\alpha \in \left(\dfrac{c}{w}, 1\right)$.

Thus, $f(\alpha, c, w)$ is concave in α.

Note that $\lim_{\alpha\to(c/w)+} f(\alpha, c, w) = w[c/w + (c/w)\ln(c/w)/(1 - c/w)] < 0$, $\lim_{\alpha\to1-} f(\alpha, c, w) = 0$, and $\lim_{\alpha\to1-} \partial f/\partial\alpha = -1/2 < 0$. Since $f(\alpha, c, w)$ is continuous and concave in α, $f(\alpha, c, w)$ crosses zero only once from below when varying α. That is, there must exist an $\alpha^*(c, w)$ such that when $\alpha \in$ ($\alpha^*(c, w)$, 1), $f(\alpha, c, w) > 0$, which means $E(p_H) > E(p_L)$. Here, $\alpha^*(c, w)$ is defined by $f(\alpha^*, c, w) = 0$, which yields Equation 3.

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