Price Competition and Endogenous Valuation in Search Advertising

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Web Appendix A

Heterogeneous Consumer Valuation

In the baseline model, we assumed that consumers have the same willingness-to-pay to avoid unnecessary distraction from the demand factor. Now we relax this assumption by considering heterogeneous consumer valuation and allow consumers' willingness-to-pay to be different from one to another. As we will see, the main results continue to hold.

We assume that consumers' willingness-to-pay satisfies a distribution with a support $[0, v]$. Correspondingly, the demand function is thus $D(p)$, where $p \in [0, v]$ with $D(0) = 1$ and $D(v) = 0$. Technically, we assume that $D(\cdot)$ is twice-continuously differentiable and non-increasing, and the revenue function $pD(p)$ is concave. Denote $p^m = \arg\max_{p \in [0,v]} pD(p)$ as the optimal monopoly price for H. We let $p^m > c$ to rule out trivial cases; otherwise, H's optimal monopoly price is below L's marginal cost, which leads to a simple pure strategy for H regardless of its winning position.

We can conduct a similar analysis as in the baseline model to derive the equilibrium. Again, no pure-strategy equilibrium exists in the second stage price competition. When H wins the first position, as in the baseline model, the two firms randomize their prices over a common support $[\underline{p}, \bar{p}]$ with a possible mass point at the upper bound. The upper bound now is H's optimal monopoly price p^m, since H has no incentive to charge a higher price given the demand function (i.e., $\bar{p} = p^m$). By charging the lower bound price $\underline{p}$, H can attract

all consumers and earn the same profit as charging the upper bound price. Therefore, if we define p^l as the solution to $p^l D(p^l) = \alpha p^m D(p^m)$, then $\underline{p} = p^l$. Given the concavity of the revenue function, both p^m and p^l are well defined. As in the baseline model, one exception from the above common-support mixed strategy equilibrium is that when the cost advantage is dominating, H may play a pure strategy (while L still plays a mixed strategy). H is better off playing a pure strategy of simply charging a price equal to L's marginal cost to occupy the entire market, if doing so generates more revenue than randomizing its price; that is, if $cD(c) > \alpha p^m D(p^m)$, or $c > p^l$ by the definition of p^l. Combining the two cases together, the two firms' pricing strategies are characterized by the following cumulative distribution functions:

$$
\begin{aligned}
F_H(p) &= \begin{cases} 1 - \frac{D(p)(p-c)}{D(\underline{p})(\underline{p}-c)} & p \in [\underline{p}, \bar{p}) \\ 1 & p = \bar{p} \end{cases} \\
F_L(p) &= \begin{cases} \frac{D(p)p - D(\underline{p})\underline{p}}{(1-\alpha)D(p)p} & p \in [\underline{p}, \bar{p}) \\ 1 & p = \bar{p} \end{cases}
\end{aligned}
\tag{18}
$$

where $\bar{p} = p^m$ and $\underline{p} = \max\{p^l, c\}$. The expected profits are $\pi_H^1 = D(\underline{p})\underline{p}$ and $\pi_L^2 = (1 - \alpha)D(\underline{p})(\underline{p} - c)$, respectively.

Similarly, when L wins the first position, we also have a support of the price distribution $[\underline{p}', \bar{p}']$. The upper bound price L may charge is its optimal monopoly price (i.e., $\bar{p}' = \arg\max_{p \in [0,v]} D(p)(p-c)$). The lower bound price is the one that attracts all consumers and leads to the same profit (i.e., $\underline{p}'$ solves $D(\underline{p}')(\underline{p}' - c) = \alpha D(\bar{p}')(\bar{p}' - c)$). The two firms adopt the following strategies:

$$
\begin{aligned}
F_H(p) &= \frac{D(p)(p-c) - D(\underline{p}')(\underline{p}'-c)}{(1-\alpha)D(p)(p-c)} & p \in [\underline{p}', \bar{p}'] \\
F_L(p) &= \begin{cases} 1 - \frac{D(\underline{p}')\underline{p}'}{D(p)p} & p \in [\underline{p}', \bar{p}') \\ 1 & p = \bar{p}' \end{cases}
\end{aligned}
\tag{19}
$$

and the resulting expected profits are $\pi_H^2 = (1-\alpha)D(\underline{p}')\underline{p}'$ and $\pi_L^1 = D(\underline{p}')(\underline{p}'-c)$, respectively.

The equilibrium outcomes (e.g., equilibrium bidding and price dispersion) can be simi-

larly derived as in the baseline model. Figure 13 illustrates the equilibrium outcome with uniformly distributed consumer valuations (i.e., $D(p) = 1 - \frac{p}{v}$, $p \in [0, v]$). Similar to the baseline case, while the prominent position is always desirable for the low-type firm, the high-type firm may find the less-prominent position more profitable in some cases (shadowed region in Figure 13a). Consequently, the high-type firm bids aggressively to win the prominent position only when either its competitive advantage or the location prominence difference is salient (unshadowed region in Figure 13b); otherwise, the low-type firm wins the prominent position (shadowed region in Figure 13b). Regarding the expected prices from the two locations, a similar pattern is observed: The expected price from the prominent position will be higher (shadowed region in Figure 13c), unless firms' competence difference overrides the location prominence difference (unshadowed region in Figure 13c).

(a) Endogenous Valuation   (b) Bidding Outcome   (c) Price Dispersion
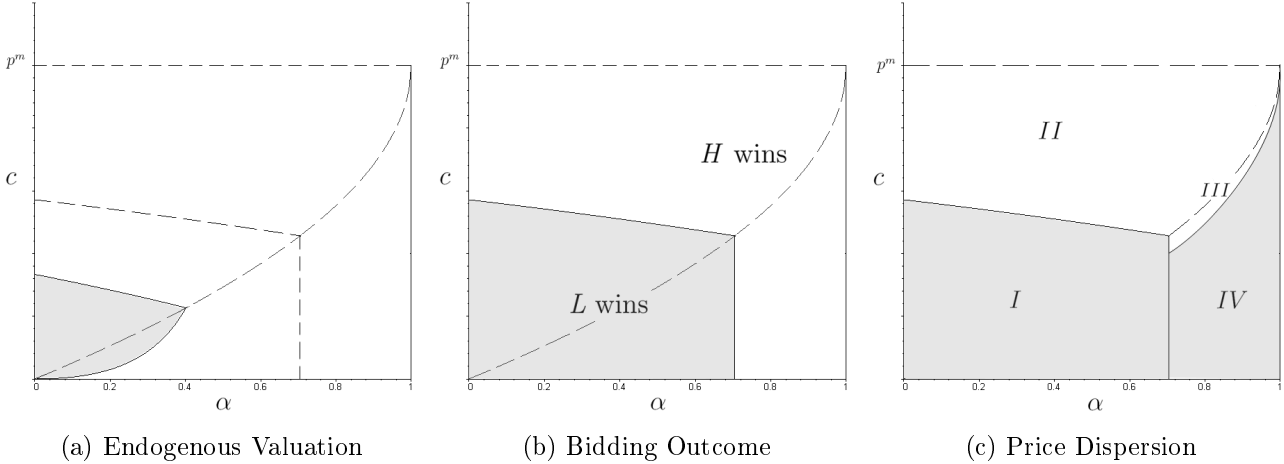
Figure 13: Uniformly Distributed Consumer Valuations

Heterogeneity in consumer valuation affects the firms' pricing decision only in that it induces a trade-off between the profit margin and the demand quantity (as in a standard monopoly pricing setting) facing both firms; yet, it does not alter the relative competitive situation between firms and thus has little influence on the major insights we have already

obtained. When reflected in the graph, its effect can be depicted as a distortion in shape with no change in pattern.

Web Appendix B

Strategic Choice of Ordering: A Brief Analysis

The following analysis mainly focuses on the case with a relatively small portion of sophisticated consumers (i.e., $\beta < \frac{1}{2}$). For the case where $\beta > \frac{1}{2}$, similar analysis applies.

Along the line of backward induction, we start with the second-stage price competition. Notice that in the second stage, sophisticated consumers can only *expect* firms' equilibrium price distributions rather than actually *observe* them. Therefore, by the concept of rational-expectations equilibrium, firms decide their pricing strategies *given* sophisticated consumers' ordering choice (which is in turn rational given firms' equilibrium pricing strategies). It thus implies that a firm's total payment to the auctioneer (i.e., a unit amount times the number of consumer visits) is considered as fixed when it makes the price decision. Therefore, we can derive the equilibrium pricing independent of the first-stage bids.

Again, no pure-strategy equilibrium exists in the second-stage price competition, and firms play mixed strategies in equilibrium. First, we consider the case in which H wins the first position. By Lemma 1, firms' equilibrium pricing strategies follow Eq.(1), given that all consumers start searching from the first position (i.e., $\sigma = 1$). Notice that $\sigma = 1$ holds in equilibrium if $E(p^1) \leq E(p^2)$. Recall that by Proposition 3, when $\alpha < \alpha^*(c, w)$, $E(p^1) < E(p^2)$, where $\alpha^*(c, w)$ is defined by Eq.(3). Therefore, when $\alpha < \alpha^*(c, w)$, all consumers starting searching from the first position and both firms pricing according to Eq.(1) are an equilibrium in the second stage.

When $\alpha > \alpha^*(c, w)$, however, $E(p^1) > E(p^2)$, and sophisticated consumers thus will not start from the first position. Assume that all sophisticated consumers start sampling from the second position (i.e., $\sigma = 0$). Then, the mixed-strategy equilibrium in pricing can be derived as

$$F_H^1(p) = \frac{[1-\alpha(1-\beta)](p-\underline{p})}{(1-\alpha)(p-c)} \quad p \in [\underline{p}, w)$$

$$F_L^2(p) = \frac{(1-\alpha\beta)(p-\underline{p})}{(1-\alpha)p} \qquad p \in [\underline{p}, w] \tag{20}$$

where $\underline{p} = \frac{\alpha(1-\beta)}{(1-\alpha\beta)}w$. Notice that H has a mass point at the upper bound as long as $c < \frac{\alpha(1-2\beta)}{1-\alpha\beta}w$. For the above to be an equilibrium, we need to ensure that $E(p^1) \geq E(p^2)$. According to $E(p^1) = E(p^2)$, we can define $\hat{\alpha}(c, w; \beta)$ as

$$\frac{\hat{\alpha}(1-2\beta)}{1-\hat{\alpha}\beta} + \frac{[1-\hat{\alpha}(1-\beta)](\frac{\hat{\alpha}(1-\beta)}{(1-\hat{\alpha}\beta)} - \frac{c}{w})}{1-\hat{\alpha}} \ln \frac{1 - \frac{c}{w}}{\frac{\hat{\alpha}(1-\beta)}{(1-\hat{\alpha}\beta)} - \frac{c}{w}} - \frac{\hat{\alpha}(1-\beta)}{1-\hat{\alpha}} \ln \frac{(1-\hat{\alpha}\beta)}{\hat{\alpha}(1-\beta)} = 0 \tag{21}$$

Notice that $\alpha^*(c, w)$ in Eq.(3) can be rewritten as $\hat{\alpha}(c, w; 0)$. As can be verified, when $\alpha > \hat{\alpha}(c, w; \beta)$, $E(p^1) > E(p^2)$. Therefore, when $\alpha > \hat{\alpha}(c, w; \beta)$, sophisticated consumers starting from the second position and firms pricing according to Eq.(20) are an equilibrium in the second stage.

In the region where $\hat{\alpha}(c, w; 0) < \alpha < \hat{\alpha}(c, w; \beta)$, as can be concluded from the above analysis, there exists no equilibrium in which sophisticated consumers play pure strategy. Instead, sophisticated consumers play mixed strategy with $0 < \sigma < 1$, and the expected prices from the two positions are equal (i.e., $E(p^1) = E(p^2)$). For any $\beta' \in (0, \beta)$, we can define a curve $\alpha = \hat{\alpha}(c, w; \beta')$ according to Eq.(21). When $(\alpha, \frac{c}{w})$ lies on that curve, sophisticated consumers playing mixed strategy $\sigma = 1 - \frac{\beta'}{\beta}$, and firms pricing the following strategies are an equilibrium in the second stage:

$$F_H^1(p) = \frac{[1-\alpha(1-\beta')](p-\underline{p})}{(1-\alpha)(p-c)} \quad p \in [\underline{p}, w)$$

$$F_L^2(p) = \frac{(1-\alpha\beta')(p-\underline{p})}{(1-\alpha)p} \qquad p \in [\underline{p}, w] \tag{22}$$

where $\underline{p} = \frac{\alpha(1-\beta')}{(1-\alpha\beta')}w$.

We next consider the case where L wins the first position. Proposition 3 indicates $E(p_L^1) > E(p_H^2)$, given that all consumers start searching from the first position. Therefore, $\sigma = 1$ cannot be part of an equilibrium. We then assume that all sophisticated consumers start

sampling from the second position ($\sigma = 0$). The resulting pricing strategies are

$$
\begin{aligned}
F_H^2(p) &= \frac{(1-\alpha\beta)(p-\underline{p})}{(1-\alpha)(p-c)} & p &\in [\underline{p}, w] \\
F_L^1(p) &= \frac{[1-\alpha(1-\beta)](p-\underline{p})}{(1-\alpha)p} & p &\in [\underline{p}, w)
\end{aligned}
\tag{23}
$$

where $\underline{p} = \frac{\alpha(1-\beta)(w-c)}{1-\alpha\beta} + c$. As we can show, F_L first order stochastically dominates F_H such that $E(p_L^1) > E(p_H^2)$, which is consistent with $\sigma = 0$. Thus, in the second-stage game when L wins the first position, the equilibrium is that sophisticated consumers start from the second position and the two firms price according to Eq.(23).

We can summarize firms' equilibrium expected profits from sales in different positions within different parameter regions as follows:

$$
(\pi_H^1, \pi_L^2) = \begin{cases}
(c, 0) & \text{if } 0 < \alpha < \frac{c}{w}; \\
(\alpha w, (1-\alpha)(\alpha w - c)) & \text{if } \frac{c}{w} < \alpha < \hat{\alpha}(c, w; 0); \\
\left(\alpha(1-\beta)w, [1-\alpha(1-\beta)](\frac{\alpha(1-\beta)}{(1-\alpha\beta)}w - c)\right) & \text{if } \hat{\alpha}(c, w; \beta) < \alpha < 1; \\
\left(\alpha(1-\beta')w, [1-\alpha(1-\beta')](\frac{\alpha(1-\beta')}{(1-\alpha\beta')}w - c)\right) & \text{if } \alpha = \hat{\alpha}(c, w; \beta'), \forall \beta' \in (0, \beta).
\end{cases}
$$

$$
(\pi_H^2, \pi_L^1) = \left([1-\alpha(1-\beta)](\frac{\alpha(1-\beta)(w-c)}{1-\alpha\beta} + c), \alpha(1-\beta)(w-c)\right)
\tag{24}
$$

For the first-stage bidding, notice that here the weighting factors (or the expected clicks at the first position) may not equal 1. For example, in the parameter region III in Figure 5c, $\omega_H = 1$ and $\omega_L = 1 - \alpha\beta$ because when L wins the first position, its equilibrium price expectation is higher so that sophisticated non-shoppers will visit the second position directly. Nevertheless, as is shown in the paper, firms' weakly dominant strategy is to submit a per-click bid $b_i = \frac{\max\{\Delta\pi_i, 0\}}{\omega_i}$, where $\Delta\pi_i = \pi_i^1 - \pi_i^2$ and $i \in \{H, L\}$, and the auctioneer ranks the firms according to the score $s_i = \omega_i b_i = \max\{\Delta\pi_i, 0\}$, which is independent of ω_i.

By comparing $\Delta\pi_H$ and $\Delta\pi_L$ (and then checking whether the winning bids are positive), we can derive the bidding outcome and pin down the boundaries of firms' winning regions. Use Figure 5c for illustration. When $0 < \alpha < \frac{c}{w}$, $\Delta\pi_H > \Delta\pi_H$ if $\frac{c}{w} > \frac{2-\alpha}{3-\alpha-\alpha\beta}$ (region II).

When $\frac{c}{w} < \alpha < \hat{\alpha}(c, w; 0)$, comparing $\Delta\pi_H$ and $\Delta\pi_L$ leads to the boundary of region III: $\frac{c}{w} = \frac{\alpha\beta(1-\alpha)(2-\alpha)}{\alpha^2(1+\beta-\beta^2)-\alpha(4-\beta)+2}$. Note that the three curves $\frac{c}{w} = \frac{2-\alpha}{3-\alpha-\alpha\beta}$, $\frac{c}{w} = \frac{\alpha\beta(1-\alpha)(2-\alpha)}{\alpha^2(1+\beta-\beta^2)-\alpha(4-\beta)+2}$, and $\frac{c}{w} = \alpha$ intersect at one point. When $\hat{\alpha}(c, w; \beta) < \alpha < 1$, $\Delta\pi_H > \Delta\pi_H$ if $\alpha > \frac{(2-\beta)-\sqrt{5\beta^2-6\beta+2}}{(1-\beta)(1+2\beta)}$ (region IV). When $\hat{\alpha}(c, w; 0) < \alpha < \hat{\alpha}(c, w; \beta)$, for any curve $\alpha = \hat{\alpha}(c, w; \beta')$ (with $\beta' \in (0, \beta)$) in between, its intersection with $\frac{c}{w} = \frac{\alpha(2-\alpha)(1-\alpha)(\beta-\beta')}{(1-\alpha\beta')[(1+\beta-\beta^2-\beta\beta')\alpha^2-(4-\beta-\beta')\alpha+2]}$ defines a cutoff in comparing $\Delta\pi_H$ and $\Delta\pi_L$. The boundary of region V consists of all these cutoffs. Together with the discussion on the expected prices in the second-stage pricing game, the spatial price dispersion pattern can be summarized as by Figure 5c.

Web Appendix C

Equilibrium Pricing in the Three-Firms Case

In this section, we provide a complete description of the equilibrium pricing strategies in the case of three competing firms. We organize the results by different parameter regions under different scenarios. The proofs to the first two results are outlined, and the rest can be analyzed in a similar fashion. For convenience, denote $H - L - L$ as the case when H stays in the first position and two L firms are in the second and third positions. Similar interpretations apply to $L - H - L$ and $L - L - H$. Also, we call the firm in position i as firm i, where $i = 1, 2, 3$. As before, we use $F_i(\cdot)$ to represent the cumulative distribution function (cdf) for the pricing strategy of firm i.

(1) The Case of $H - L - L$

Result 1. *In the case of $H - L - L$, when $\frac{c}{w} < \frac{\alpha^2}{1+\alpha(1-\alpha)}$, the equilibrium pricing strategies*

are:

$$
F_1(p) = \begin{cases} 1 - \frac{\bar{p}_2 - c}{p - c} & p \in [\bar{p}_2, \bar{p}_1) \\[2mm] 1 & p = \bar{p}_1 \end{cases}
$$

$$
F_2(p) = \begin{cases} 1 - \frac{\bar{p}_3 - c}{p - c} & p \in [\bar{p}_3, \bar{p}_2) \\[2mm] 1 - \frac{(\bar{p}_1 - p)}{(1 - \alpha)p} & p \in [\bar{p}_2, \bar{p}_1] \end{cases}, \tag{25}
$$

$$
F_3(p) = 1 - \frac{\alpha(\bar{p}_2 - p)}{(1 - \alpha)(p - c)} \quad p \in [\bar{p}_3, \bar{p}_2],
$$

where $\bar{p}_1 = w$, $\bar{p}_2 = \frac{1}{1 + \alpha(1 - \alpha)}\bar{p}_1$, *and* $\bar{p}_3 = \alpha(\bar{p}_2 - c) + c$.

For convenience, we use the superscript "$+$" to denote the cdf in the upper half of the support, and we use the superscript "$-$" as the cdf in the lower half of the support. For example, in Eq.25, we refer to $F_2(p) = 1 - \frac{(\bar{p}_1 - p)}{(1 - \alpha)p}$ for $p \in [\bar{p}_2, \bar{p}_1]$ as $F_2^+(p)$ and refer to $F_2(p) = 1 - \frac{\bar{p}_3 - c}{p - c}$ for $p \in [\bar{p}_3, \bar{p}_2)$ as $F_2^-(p)$.

First, we show that $F_i(\cdot)$ $(i = 1, 2, 3)$ are well-defined cumulative distribution functions. Notice that $c < \bar{p}_3 < \bar{p}_2 < \bar{p}_1 = w$, so that the supports are well defined. Also, we can check all the bounds: $F_1(\bar{p}_2) = 0$, $F_1^-(\bar{p}_1) = 1 - \frac{\bar{p}_2 - c}{\bar{p}_1 - c} < 1$, $F_2(\bar{p}_3) = 0$, and $F_2^-(\bar{p}_2) = F_2^+(\bar{p}_2)$ because $\frac{\bar{p}_3 - c}{\bar{p}_2 - c} = \frac{(\bar{p}_1 - \bar{p}_2)}{(1 - \alpha)\bar{p}_2} = \alpha$, $F_2(\bar{p}_1) = 1$, $F_3(\bar{p}_3) = 1 - \frac{\alpha(\bar{p}_2 - \bar{p}_3)}{(1 - \alpha)(\bar{p}_3 - c)} = 1 - \frac{\alpha(1 - \alpha)(\bar{p}_2 - c)}{(1 - \alpha)(\bar{p}_3 - c)} = 0$, and $F_3(\bar{p}_2) = 1$. Moreover, all $F_i(p)$ are increasing in p. Therefore, all $F_i(\cdot)$ are well-defined *cdf*s.

Second, we can show that each position i yields a constant expected profit π_i within the entire support, $i = 1, 2, 3$. For example, consider the second position. For $p \in [\bar{p}_3, \bar{p}_2)$,

$$
\pi_2^-(p) = \alpha(1 - \alpha)(p - c) + (1 - \alpha)^2[1 - F_3(p)](p - c), \tag{26}
$$

where the first part on the right-hand side accounts for the demand from those consumers who stop searching at the second position (they purchase from position 2 for sure because p is lower than the lower bound of F_1's support, $\bar{p}_2$), and the second part accounts for the

demand from shoppers when its price is lower than the third position. Substituting in $F_3(p)$, we have:

$$\begin{aligned}
\pi_2^-(p) &= \alpha(1-\alpha)(p-c) + (1-\alpha)^2 \frac{\alpha(\bar{p}_2 - p)}{(1-\alpha)(p-c)}(p-c) \\
&\equiv \alpha(1-\alpha)(\bar{p}_2 - c).
\end{aligned} \tag{27}$$

Similarly, for $p \in [\bar{p}_2, \bar{p}_1]$, we have:

$$\begin{aligned}
\pi_2^+(p) &= \alpha(1-\alpha)[1 - F_1(p)](p-c) \\
&= \alpha(1-\alpha)\frac{\bar{p}_2 - c}{p-c}(p-c) \equiv \alpha(1-\alpha)(\bar{p}_2 - c).
\end{aligned} \tag{28}$$

Therefore, firm 2 achieves a constant expected profit level $\pi_2 = \alpha(1-\alpha)(\bar{p}_2 - c)$ when charging any price within its support $[\bar{p}_3, \bar{p}_1]$. Similar analysis applies for both $\pi_1 = \alpha\bar{p}_1$ and $\pi_3 = (1-\alpha)^2(\bar{p}_3 - c)$.

Finally, we need to verify that there is no profitable unilateral deviation. For example, if H deviates to price $p \in [\bar{p}_3, \bar{p}_2)$, the profit function would then become:

$$\begin{aligned}
\pi_1'(p) &= \alpha p + \alpha(1-\alpha)p\left[1 - F_2^-(p)\right] + (1-\alpha)^2 p\left[1 - F_2^-(p)\right][1 - F_3(p)] \\
&= \alpha p + \frac{\bar{p}_3 - c}{p-c}\alpha(1-\alpha)(\bar{p}_2 - c)\frac{p}{p-c},
\end{aligned} \tag{29}$$

which is convex in p. Notice that when $\frac{c}{w} < \frac{\alpha^2}{1+\alpha(1-\alpha)}$, $\pi_1(\bar{p}_2) \geq \pi_1'(\bar{p}_3)$. Therefore, $\pi_1'(p) < \pi_1(\bar{p}_2)$ for $\forall p \in [\bar{p}_3, \bar{p}_2)$. Thus, when $\frac{c}{w} < \frac{\alpha^2}{1+\alpha(1-\alpha)}$, H will not deviate to charge a price below the given support. Similarly, we can show that it is not profitable for firm 3 to charge a price above its given support.

Altogether, we can conclude that the price strategy described is a mixed-strategy equilibrium.

Result 2. *In the case of $H-L-L$, when $\frac{\alpha^2}{1+\alpha(1-\alpha)} < \frac{c}{w} < \min\{\alpha, \frac{1-\alpha(1-\alpha)}{[1+\alpha(1-\alpha)]^2}\}$, the equilibrium*

pricing strategies are:

$$F_1(p) = \begin{cases} F_1^*(p) & p \in [\bar{p}_4, \bar{p}_3] \\ 1 - \frac{\alpha \bar{p}_1 - c}{\alpha(p-c)} & p \in [\bar{p}_2, \bar{p}_1) \\ 1 & p = \bar{p}_1 \end{cases}$$

$$F_2(p) = \begin{cases} F_2^*(p) & p \in [\bar{p}_4, \bar{p}_3) \\ 1 - \frac{\alpha(\bar{p}_2 - c)}{p-c} & p \in [\bar{p}_3, \bar{p}_2) \\ 1 - \frac{(\bar{p}_1 - p)}{(1-\alpha)p} & p \in [\bar{p}_2, \bar{p}_1] \end{cases} \tag{30}$$

$$F_3(p) = \begin{cases} F_3^*(p) & p \in [\bar{p}_4, \bar{p}_3) \\ 1 - \frac{\alpha(\bar{p}_2 - p)}{(1-\alpha)(p-c)} & p \in [\bar{p}_3, \bar{p}_2], \end{cases}$$

where $\bar{p}_1 = w$, $\bar{p}_2 = \frac{1}{1+\alpha(1-\alpha)}w$, $\bar{p}_4 = \alpha w$, and $\bar{p}_3$ is the solution between $\bar{p}_4$ and $\bar{p}_2$ to the equation:

$$[1 + \alpha(1-\alpha)]\bar{p}_3^2 - [\alpha(1-\alpha)(1+2c) + 2c]\bar{p}_3 + [1 + \alpha(1-\alpha)]^2 c^2 = 0, \tag{31}$$

and $\{F_1^*(p), F_2^*(p), F_3^*(p)\}$ solves:

$$\begin{cases} \alpha p + \alpha(1-\alpha)p[1 - F_2^*(p)] + (1-\alpha)^2 p[1 - F_2^*(p)][1 - F_3^*(p)] = \alpha w \\ \alpha(1-\alpha)(p-c)[1 - F_1^*(p)] + (1-\alpha)^2(p-c)[1 - F_1^*(p)][1 - F_3^*(p)] = (1-\alpha)(\alpha w - c) \\ (1-\alpha)^2(p-c)[1 - F_1^*(p)][1 - F_2^*(p)] = (1-\alpha)^2(\alpha w - c). \end{cases}$$
$$\tag{32}$$

Following similar analysis, we can show that the strategy is indeed an equilibrium, as is briefly outlined below.

First, the supports and the *cdf*s are well defined. Notice that when $\frac{\alpha^2}{1+\alpha(1-\alpha)} < \frac{c}{w}$, $F_1(\bar{p}_2) = 1 - \frac{\alpha \bar{p}_1 - c}{\alpha(\bar{p}_2 - c)} > 0$, which indicates that H may charge a price lower than $\bar{p}_2$ with pos-

itive probability. Also, notice that the parameter region $\frac{\alpha^2}{1+\alpha(1-\alpha)} < \frac{c}{w} < \min\{\alpha, \frac{1-\alpha(1-\alpha)}{[1+\alpha(1-\alpha)]^2}\}$ ensures that there is a unique solution to Eq.(31) between $\bar{p}_4$ and $\bar{p}_2$. In fact, if we denote the right-hand side of Eq.(31) as $g(\bar{p}_3)$, under the given parameter condition, $g(\bar{p}_4) < 0$ and $g(\bar{p}_2) > 0$, which ensures that $\bar{p}_3$ is well defined. Moreover, notice that $\bar{p}_3$ actually solves $F_1^*(\bar{p}_3) = F_1(\bar{p}_2)$. To see this, by substituting $p = \bar{p}_3$ and $F_1^*(\bar{p}_3) = 1 - \frac{\alpha\bar{p}_1 - c}{\alpha(\bar{p}_2 - c)}$ into Eq.(32), we can then solve the last two equations of Eq.(32) together and get:

$$
\begin{cases}
F_2^*(\bar{p}_3) = 1 - \frac{\alpha(\bar{p}_2 - c)}{\bar{p}_3 - c} \\
F_3^*(\bar{p}_3) = 1 - \frac{\alpha(\bar{p}_2 - \bar{p}_3)}{(1-\alpha)(\bar{p}_3 - c)}.
\end{cases}
\tag{33}
$$

Substituting back into the first equation of Eq.(32), we have Eq.(31).

Second, firms achieve constant profit within their price supports: (i) Notice that the left-hand sides in Eq.(32) are in fact the profit functions for the three firms when charging $p \in [\bar{p}_4, \bar{p}_3]$, and the right-hand sides are the constant expected profit they achieve over their entire price supports. Therefore, Eq.(32) ensures that the three firms all achieve a constant profit level when pricing $p \in [\bar{p}_4, \bar{p}_3]$. (ii) $F_1(\bar{p}_3) = F_1(\bar{p}_2)$ indicates that F_1 does not put any mass over the interval $(\bar{p}_3, \bar{p}_2)$, which means that H does not charge any price between $\bar{p}_3$ and $\bar{p}_2$. Thus, for $p \in (\bar{p}_3, \bar{p}_2)$, firm 2 and firm 3 achieve a constant profit:

$$
\begin{cases}
\pi_2(p) = \alpha(1-\alpha)(p-c)[1-F_1(\bar{p}_2)] + (1-\alpha)^2(p-c)[1-F_1(\bar{p}_2)][1-F_3(p)] \equiv (1-\alpha)(\alpha w - c) \\
\pi_3(p) = (1-\alpha)^2(p-c)[1-F_1(\bar{p}_2)][1-F_2^*(p)] \equiv (1-\alpha)^2(\alpha w - c).
\end{cases}
\tag{34}
$$

(iii) $F_3(\bar{p}_2) = 1$, indicating that firm 3 does not charge any price above $\bar{p}_2$. Therefore, for $p \in (\bar{p}_2, \bar{p}_1)$,

$$
\begin{cases}
\pi_1(p) = \alpha p + \alpha(1-\alpha)p[1-F_2(p)] \equiv \alpha w \\
\pi_2(p) = \alpha(1-\alpha)(p-c)[1-F_1(p)] \equiv (1-\alpha)(\alpha w - c).
\end{cases}
\tag{35}
$$

Finally, as we can check, firm 1 pricing $p \in (\bar{p}_3, \bar{p}_2)$ and firm 3 pricing $p \in (\bar{p}_2, \bar{p}_1)$ both result in lower profits. Therefore, there is no profitable deviation. As a result, the pricing strategy described is a mixed-strategy equilibrium.

Result 3. *In the case of $H - L - L$, when $\min\{\alpha, \frac{1-\alpha(1-\alpha)}{[1+\alpha(1-\alpha)]^2}\} < \frac{c}{w} < \alpha$, the equilibrium pricing strategies are:*

$$
F_1(p) = \begin{cases} F_1^*(p) & p \in [\bar{p}_3, \bar{p}_2) \\[2mm] 1 - \frac{\alpha \bar{p}_1 - c}{\alpha(p-c)} & p \in [\bar{p}_2, \bar{p}_1) \\[2mm] 1 & p = \bar{p}_1 \end{cases}
$$

$$
F_2(p) = \begin{cases} F_2^*(p) & p \in [\bar{p}_3, \bar{p}_2) \\[2mm] 1 - \frac{(\bar{p}_1 - p)}{(1-\alpha)p} & p \in [\bar{p}_2, \bar{p}_1] \end{cases} \tag{36}
$$

$$
F_3(p) = F_3^*(p) \quad p \in [\bar{p}_3, \bar{p}_2],
$$

where $\bar{p}_1 = w$, $\bar{p}_2 = \frac{1}{1+\alpha(1-\alpha)}w$, and $\bar{p}_3 = \alpha w$, and $\{F_1^*(p), F_2^*(p), F_3^*(p)\}$ solves Eq.(32).

The analysis in this case is similar to that for Result 2.

Result 4. *In the case of $H - L - L$, when $\frac{c}{w} > \alpha$, firm 1 pricing $p_1 = c$, firm 3 pricing $p_3 = w$, and firm 2 pricing according to:*

$$
F_2(p) = \begin{cases} 1 - \frac{c-\alpha p}{(1-\alpha)p} & p \in [c, w) \\[2mm] 1 & p = w \end{cases}, \tag{37}
$$

is an equilibrium.

This is a trivial case in which the cost advantage is so significant that H simply charges c and both L firms achieve zero profit. Firm 2 prices in a way that firm 1 has no profitable deviation.

(2) The Case of $L - H - L$

Result 5. *In the case of $L - H - L$, when $\frac{c}{w} < \alpha$, the equilibrium pricing strategies are:*

$$
F_1(p) = \begin{cases} 1 - \frac{\bar{p}_2}{p} & p \in [\bar{p}_2, \bar{p}_1) \\ 1 & p = \bar{p}_1 \end{cases}
$$

$$
F_2(p) = \begin{cases} 1 - \frac{\bar{p}_3 - c}{p - c} & p \in [\bar{p}_3, \bar{p}_2) \\ 1 - \frac{(\bar{p}_1 - p)}{(1-\alpha)(p-c)} & p \in [\bar{p}_2, \bar{p}_1] \end{cases} \tag{38}
$$

$$
F_3(p) = 1 - \frac{\alpha(\bar{p}_2 - p)}{(1-\alpha)p} \quad p \in [\bar{p}_3, \bar{p}_2] ,
$$

where $\bar{p}_1 = w$, $\bar{p}_2 = \frac{\bar{p}_1 + (1-\alpha)c}{1 + \alpha(1-\alpha)}$, and $\bar{p}_3 = \alpha_2 \bar{p}_2$.

This case can be analyzed similarly to the analysis of Result 1.

Result 6. *In the case of $L - H - L$, when $\frac{c}{w} > \alpha$, firm 1 pricing $p_1 = w$, firm 2 pricing $p_2 = c$, and firm 3 pricing according to:*

$$
F_3(p) = \begin{cases} 1 - \frac{c - \alpha p}{(1-\alpha)p} & p \in [c, w) \\ 1 & p = w \end{cases} , \tag{39}
$$

is an equilibrium.

This is a trivial case, similar to Result 4.

(3) The Case of $L - L - H$

Result 7. *In the case of $L - L - H$, the equilibrium pricing strategies are:*

$$
F_1\left(p\right) \; = \; \begin{cases} 1 - \frac{\bar{p}_2 - c}{p - c} & p \in [\bar{p}_2, \bar{p}_1) \\[2mm] 1 & p = \bar{p}_1 \end{cases}
$$

$$
F_2\left(p\right) \; = \; \begin{cases} 1 - \frac{\bar{p}_3}{p} & p \in [\bar{p}_3, \bar{p}_2) \\[2mm] 1 - \frac{(\bar{p}_1 - p)}{(1-\alpha)(p-c)} & p \in [\bar{p}_2, \bar{p}_1] \end{cases} \tag{40}
$$

$$
F_3\left(p\right) \; = \; 1 - \frac{\alpha(\bar{p}_2 - p)}{(1-\alpha)(p-c)} \quad p \in [\bar{p}_3, \bar{p}_2] \; ,
$$

where $\bar{p}_1 = w$, $\bar{p}_3 = \alpha\left(\bar{p}_2 - c\right) + c$, and

$$
\bar{p}_2 = \frac{-\left(1-\alpha\right)\left(1-2\alpha\right)c + w + \sqrt{5\left(1-\alpha\right)^2 c^2 - 2\left(1-\alpha\right)\left(1-2\alpha\right)wc + w^2}}{2\left[1 + \alpha\left(1-\alpha\right)\right]}. \tag{41}
$$

This case can be analyzed similarly to the analysis of Result 1 and Result 5.

Web Appendix D

More Results for the Three-Firms Case

Recall the equilibrium profits in the price competition from Table 2. We denote the three firms' second-stage equilibrium profits in the case of $H - L - L$ (i.e., H wins the first position and the two L firms are in the second and third positions) as u_H^1, u_L^2, and u_L^3, respectively. Similarly, we denote the equilibrium profits in the $L - H - L$ case as v_L^1, v_H^2, and v_L^3, and we denote the equilibrium profits in the $L - L - H$ case as w_L^1, w_L^2, and w_H^3.

In deriving the bidding outcomes, we focus on the particular type of equilibria in which the two low-type firms behave symmetrically, that is, they adopt the same bidding strategies. Figure 14 summarizes the bidding outcome. In the shadowed region, which corresponds to the condition that $u_H^1 - w_H^3 < \min\{\frac{w_L^2 - v_L^3}{1-\alpha}, \frac{w_L^1 - w_L^2}{\alpha}\}$, H bids lower than the two L firms and stays in the third position. More specifically, in this region, $b_H = u_H^1 - w_H^3$ and $b_L =$

$(1-\alpha)\left(u_H^1 - w_H^3\right) + w_L^1 - w_L^2$. Notice that the two low-type firms submit the same bid b_L because they are indifferent between one position higher or not, and hence win a higher position with equal probability. Also, notice that in our setting, all three firms generate the same number of clicks when they are in the same position. Therefore, they have the same weighting factor, and the ranking is determined by the amount of their per-click bids b_i. As a result, when $u_H^1 - w_H^3 < \frac{w_L^1 - w_L^2}{\alpha}$, $b_H < b_L$, and H is therefore placed in the third position. To ensure that the bidding strategy is an equilibrium, we need to further check the possible deviations. As we can see, given b_L, H does not want to outbid L because its net profit would be $u_H^1 - b_L$, which is lower than its net profit in the third position, w_H^3. Similarly, given b_H and the other L firm's bid b_L, neither L firm has a profitable deviation: First, staying in the first position yields a net profit $w_L^1 - b_L$, which equals the net profit from the second position $w_L^2 - (1 - \alpha) b_H$ (notice that the number of click-throughs in the second position is $1 - \alpha$). Second, underbidding to stay in the third position is not optimal either because the net profit then would be v_L^3—lower than the in-equilibrium net profit $w_L^2 - (1 - \alpha) b_H$ as $b_H < \frac{w_L^2 - v_L^3}{1-\alpha}$.

On the other hand, in the unshadowed region in Figure 14 (i.e., when $u_H^1 - w_H^3 > \max\{v_L^1 - u_L^3, \frac{u_L^2 - u_L^3}{1-\alpha}\}$), H outbids both L firms and wins the first position. In this case, $b_H = u_H^1 - w_H^3$, and $b_L = \frac{u_L^2 - u_L^3}{1-\alpha}$. Following similar arguments, we can show that none of the three firms has a profitable deviation. For H, because $b_L < u_H^1 - w_H^3$, underbidding to be in the third position would lead to a lower net profit (w_H^3) than being in the first position $(u_H^1 - b_L)$. For L, staying in the second or the third position results in the same net profit because $u_L^2 - (1 - \alpha) b_L = u_L^3$. It is not profitable for L to outbid H because being in the first position yields a payoff of $v_L^1 - b_H$, which is lower than the payoff in the second or third position (u_L^3).
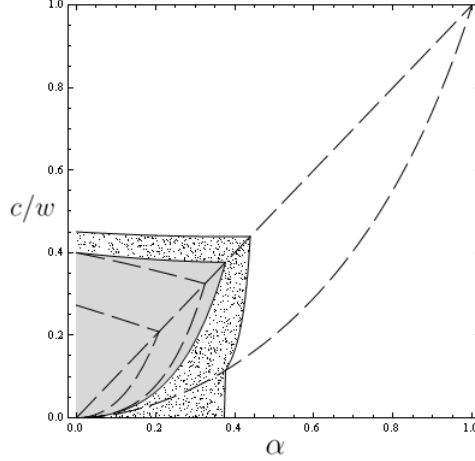
Figure 14: Bidding Outcome in the Case of Three Firms

An interesting result is found in the dotted region in Figure 14. In this region, firms adopt mixed-strategy bidding in equilibrium. We use an example to better illustrate the idea.

Example. When $\alpha = 0.4$, $c = 0.3$, and $w = 1$, in equilibrium, both L firms symmetrically bid $b_L = 0.219$ and H adopts mixed-strategy bidding. With probability $p = 0.23$, H bids as high as $b'_H = 0.352$ and wins the first position; with probability $1 - p = 0.77$, H bids as low as $b_H = 0.155$ and stays in the third position.

The example illustrates a bidding outcome in which the high-type firm switches between a top position and a lower position. In this case, $b_L = u_H^1 - w_H^3$ so that H is indifferent between attaining the first position and attaining the third one. For this reason, H is willing to mix its bid. H mixes in such a way that neither L wants to overbid or underbid its counterpart (i.e., bidding b_L is optimal, given that the other L firm also bids b_L). For this reason, the mixing probability p satisfies:

$$p = \frac{b_L - (1 - \alpha) b_H - w_L^1 + w_L^2}{(u_L^2 - u_L^3) - (w_L^1 - w_L^2) - (1 - \alpha) b_H + \alpha b_L}. \tag{42}$$

In addition, H's high bid $b'_H = (1 - \alpha) b_L + v_L^1 - u_L^2$ is high enough that L will not deviate

to bid higher than b'_H; meanwhile, H's low bid $b_H = \frac{w_L^2 - v_L^3}{1-\alpha}$ is low enough that bidding lower than b_H is not optimal for L either. As a result, $b_H < b_L < b'_H$ in equilibrium.

As we can see from Figure 14, the dotted region with mixed-strategy bidding serves as a natural transition between the two deterministic cases that involves only pure-strategy bidding.
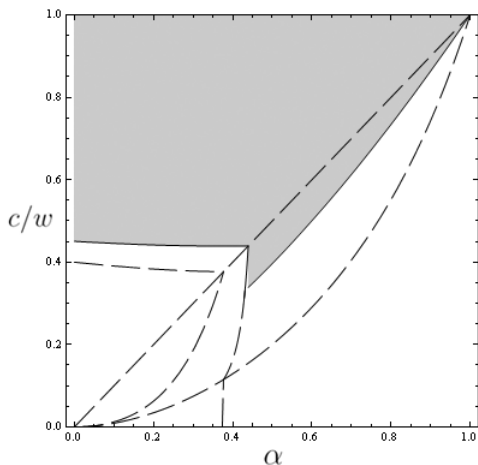

Figure 15: Price Dispersion in the Case of Three Firms

Figure 15 illustrates the spatial price dispersion pattern in the three-firms case. As we can see, similar results exist. In the highlighted region, the high-type firm wins the first position, and the expected price from the first position is lower than that from the second position, indicating that, depending on the endogenous competitive situation, an expensive location may not necessarily be associated with expensive products.