Effects of the Presence of Organic Listing in Search Advertising

Lizhen Xu

Jianqing Chen

Andrew Whinston

Web Appendix: The Case of Multiple Competing Firms

In this section, we extend the analysis from duopolistic competition to the case with multiple competing firms.

Model

We now consider $n\,(\geq 2)$ firms in the market. One of them sell products catering to the mainstream market, which is termed as M; the other $n-1$ firms' products are designed for particular niche markets, which are denoted as $N_1, ..., N_{n-1}$. There is a continuum of consumers with mass 1. Each consumer has a unit demand of the product. Consumers have different preferences. Following the main model, we assume θ $(0 < \theta < \frac{1}{2})$ of them are N-type consumers who prefer the niche firms' products to the mainstream firm's, and $1-\theta$ are M-type consumers who prefer the mainstream product. Among the $n-1$ niche firms, we consider the case in which niche firms develop particular features of the product that are mutually exclusive so that each consumer values one niche product only and derives zero utility for the other niche products. For ease of exposition, we assume all niche firms equally share the market preference. An N-type consumer derives utility v from consuming the niche product that she values, and derives discounted utility $\tilde{k}v$ from consuming the mainstream product. An M-type consumer derives utility v from consuming the mainstream product and discounted utility $\tilde{k}v$ from consuming the niche product that she values. $\tilde{k}$ is uniformly distributed between 0 and 1 among all consumers. We normalize v to 1 without loss of generality.

We follow a similar way in modeling consumers' click behavior on the search engine results page. The M firm is listed in a top organic position which attracts most attention (i.e., α_{i_M} can be very high). Similar as before, we let $\alpha_{i_M} = 1$ to emphasize the diminishing promotive incentive when an advertiser's organic rank is high. In other words, consumers click M's link with probability 1. In contrast, the niche firms' organic ranks are much less satisfactory and the differences among their organic exposures can be negligible compared to the difference between the mainstream's and theirs. We thus assume that the niche firms' overall attention levels are determined by their sponsored ranking, that is, $1 - \left(1 - \alpha_{i_{N_k}}\right)(1-\beta_j)\left(1 - \gamma_{i_{N_k}j}\right) = 1 - \left(1 - \alpha_{i_{N_{k'}}}\right)(1-\beta_j)\left(1 - \gamma_{i_{N_{k'}}j}\right)$ for any $k, k' \in \{1, ..., n-1\}$. More specifically, for a niche firm winning the jth sponsored slot, the combined probability of its link (either organic or sponsored) being clicked is $1 - \psi_j$. There are $m\,(\geq 2)$ sponsored slots and we only discuss the case in which $m \geq n$. Similar analysis can be applied to the case of $m < n$. Without loss of generality, we let the exposure of sponsored positions monotonically decrease from the first to the last, that is, $0 < \psi_1 < ... < \psi_n < 1$.

Analysis and Results

We can derive the equilibrium pricing similarly as in the main model. The demand function facing the niche firm staying in the jth sponsored position, given M's price p_M, can be written as

$$D_N^j(p; p_M) = \frac{1}{n-1}\left(1 - \psi_j\right) S_N\left(p, p_M\right),$$

where S_N is defined by Eq.(1). Maximizing the profit function $pD_N^j(p; p_M)$ gives the optimal pricing. Notice that the maximization problem is actually independent of ψ_j, which implies that all niche firms charge the same price in equilibrium. Given niche firms' price p_N, the demand function facing M when it stays in the kth sponsored position can be written as

$$\begin{aligned} D_M^k(p; p_N) &= \sum_{i \neq k} \frac{1}{n-1}\left[\psi_i A_M(p) + (1 - \psi_i) S_M(p, p_N)\right] \\ &= \bar{\psi}_{-k} A_M(p) + \left(1 - \bar{\psi}_{-k}\right) S_M(p, p_N), \end{aligned}$$

where $\bar{\psi}_{-k} = \frac{1}{n-1}\sum_{i\neq k}\psi_i$. Maximizing the profit functions simultaneously, we can derive the equilibrium prices (when M stays in the kth sponsored position) as follows.

$$\begin{cases} p_M^* &= \min\{\frac{2-\theta\left(1-\bar{\psi}_{-k}\right)}{3(1-\theta)\left(1-\bar{\psi}_{-k}\right)+4\theta\bar{\psi}_{-k}}, 1\} \\ p_N^* &= \frac{\theta+(1-\theta)p_M^*}{2(1-\theta)}. \end{cases}$$

Therefore, the equilibrium profit functions can be derived accordingly:

$$\begin{cases} \pi_M^k = \pi_M^*\left(\bar{\psi}_{-k}, \theta\right) \\ \pi_N^{j(k)} = \frac{1}{n-1}\left(1-\psi_j\right) f\left(\bar{\psi}_{-k}, \theta\right), \end{cases}$$

where π_M^k is the equilibrium profit for the M firm if it stays in the kth sponsored position, and $\pi_N^{j(k)}$ stands for the equilibrium profit of the N firm staying in the jth sponsored position when M gets the kth sponsored slot ($j \neq k$). Here, $\pi_M^*\left(\psi, \theta\right)$ is the same as is defined in Table 2, while $f\left(\psi, \theta\right)$ is defined as

$$f\left(\psi, \theta\right) = \begin{cases} \frac{\left[1-\theta^2+\theta(3\theta-1)\psi\right]^2}{(1-\theta)[3(1-\theta)(1-\psi)+4\theta\psi]^2} & \psi < \frac{1}{3} \\ \frac{1}{4(1-\theta)} & \psi \geq \frac{1}{3}. \end{cases}$$

Before we derive the bidding equilibrium, we first summarize a result which will be useful in the analysis of equilibrium bidding. In fact, the following result is a counterpart of Lemma 1 in the oligopolistic case. It shows that the difference between the mainstream firm's and the niche firms' incentives to improve their sponsored ranks decrease as the competence difference reduces (i.e., as θ increases).

Lemma 2. $\pi_M^*\left(\psi', \theta\right) - \pi_M^*\left(\psi''', \theta\right) - \frac{1}{n-1}\left(1-x_1\right) f\left(\psi'', \theta\right) + \frac{1}{n-1}\left(1-x_2\right) f\left(\psi', \theta\right)$ is decreasing in θ, where $1 > \psi' > \psi'' \geq \psi''' > 0$ and $1 > x_2 \geq x_1 > 0$.

To derive the bidding equilibrium, we follow Edelman et al. (2007) and consider the locally envy-free equilibrium in the generalized second-price auctions. In a locally envy-free equilibrium, any advertiser does not want to exchange bids with the one ranked one position above it in the sponsored list. We focus on the particular type of locally envy-free equilibrium studied by Edelman et al. (2007), in which each advertiser bids an amount that equals its own payment plus its own value difference between staying in the current position and moving one position up. In other words, each advertiser's payment is equal to the negative externality that it imposes on all the other advertisers. The equilibrium analysis is more complex in our setting because any change of M's position will change the values of all niche firms in all positions.

We are particularly interested in two types of equilibria, namely, when the mainstream firm wins the first sponsored position, and when the niche firms win the first $n-1$ sponsored positions and the mainstream firm gets the last. Applying the results from Lemma 2, we show that when θ is large, the latter may hold in equilibrium, and the former can be an equilibrium when θ is small, which are similar to Proposition 1.

Proposition 8. *Generically, there exist cutoffs* $\hat{\theta}(\psi_1, ..., \psi_n)$ *and* $\tilde{\theta}(\psi_1, ..., \psi_n)$*, such that when* $0 < \theta < \hat{\theta}(\psi_1, ..., \psi_n)$*,* M *winning the first sponsored position and all niche firms staying in the second to the nth positions is an equilibrium, and when* $\tilde{\theta}(\psi_1, ..., \psi_n) < \theta < \frac{1}{2}$*, all niche firms staying in the first* $n-1$ *positions and* M *staying in the last is an equilibrium.*

Proposition 8 reveals a similar pattern in the equilibrium bidding outcomes. With reasonable market preference shares, although still weaker than the leading firm, niche firms have higher bidding incentives and get better sponsored positions. Such outcome, which is not aligned with advertisers' inherent competitive strength, reflects the interplay between organic listing and sponsored bidding, and the balance between the promotive and preventive effects. On the other hand, when niche firms are too weak, the leading firm's preventive motivation dominates, making it occupy the top sponsored position.

Given the similarity in the equilibrium bidding outcomes, the rest analysis and the main results from the duopoly case can be expected to hold qualitatively when extended to the oligopoly case. The following example illustrates the results.

Example 3. *We consider the case with* $n = 3$*,* $\psi_1 = .02$*,* $\psi_2 = .15$*, and* $\psi_3 = .20$*. As we can show, when* $\theta > .352$*, the two niche firms bidding higher than the mainstream firm is an equilibrium in the co-listing case, whereas such an equilibrium cannot arise in the benchmark case for any* θ*. If* $\theta = .4$*, for example, then* $b_N^1 > b_N^2 = .0290 > b_M^3 = .0045$ *is an equilibrium in the co-listing case, while* $b_M^1 > b_N^2 = .0366 > b_N^3 = .0098$ *is an equilibrium in the benchmark case. The co-listing case generates higher social welfare, consumer surplus, sales diversity, but lower immediate revenue for search engine than the benchmark case. More specifically,* $W^C = .9644 > W^B = .9288$*,*

$CS^C = .1308 > CS^B = .1011$, $G^C = .2288 < G^B = .2435$, *and* $IR^C = .0335 < IR^B = .0463$.

Proofs

Proof of Lemma 2. To prove that the objective function is decreasing in θ is to prove

$$\frac{\partial}{\partial\theta}\pi_M^*\left(\psi',\theta\right) - \frac{\partial}{\partial\theta}\pi_M^*\left(\psi''',\theta\right) + \frac{1}{n-1}\left(1-x_2\right)\frac{\partial}{\partial\theta}f\left(\psi',\theta\right) - \frac{1}{n-1}\left(1-x_1\right)\frac{\partial}{\partial\theta}f\left(\psi'',\theta\right) < 0. \tag{25}$$

Recall that Lemma 1 shows that $\frac{\partial^2}{\partial\psi\partial\theta}\pi_M^*\left(\psi,\theta\right) < 0$. Since $\psi' > \psi'''$, therefore, $\frac{\partial}{\partial\theta}\pi_M^*\left(\psi',\theta\right) - \frac{\partial}{\partial\theta}\pi_M^*\left(\psi''',\theta\right) < 0$, that is, the first half of the LHS of Eq.(25) is negative. Also, we can show that when $\theta \geq 0.153$, $\frac{\partial^2}{\partial\psi\partial\theta}f\left(\psi,\theta\right) \leq 0$. Notice that $\frac{\partial}{\partial\theta}f\left(\psi,\theta\right) > 0$. Therefore,

$$\left(1-x_2\right)\frac{\partial}{\partial\theta}f\left(\psi',\theta\right) - \left(1-x_1\right)\frac{\partial}{\partial\theta}f\left(\psi'',\theta\right) \leq \left(1-x_1\right)\left[\frac{\partial}{\partial\theta}f\left(\psi',\theta\right) - \frac{\partial}{\partial\theta}f\left(\psi'',\theta\right)\right] \leq 0.$$

Thus, when $\theta \geq 0.153$, the second half of the LHS of Eq.(25) is also non-positive, which implies that Eq.(25) holds.

When $\theta < 0.153$, the second half of the LHS of Eq.(25) could be positive so that we need to compare the magnitudes of the two parts. We discuss the case in which $\psi''' \leq \psi'' < \psi' \leq \frac{1}{3}$, the other cases can be easily proved. By mean value theorem, we have

$$\frac{\partial}{\partial\theta}\pi_M^*\left(\psi',\theta\right) - \frac{\partial}{\partial\theta}\pi_M^*\left(\psi''',\theta\right) = \frac{\partial^2}{\partial\psi\partial\theta}\pi_M^*\left(\hat{\psi},\theta\right)\left(\psi'-\psi'''\right),$$

where $\hat{\psi}$ is some value between ψ' and ψ'''. Similarly,

$$\begin{aligned}\frac{1}{n-1}\left(1-x_2\right)\frac{\partial}{\partial\theta}f\left(\psi',\theta\right) - \frac{1}{n-1}\left(1-x_1\right)\frac{\partial}{\partial\theta}f\left(\psi'',\theta\right) &\leq \frac{1}{n-1}\left(1-x_1\right)\left[\frac{\partial}{\partial\theta}f\left(\psi',\theta\right) - \frac{\partial}{\partial\theta}f\left(\psi'',\theta\right)\right] \\ &= \frac{1}{n-1}\left(1-x_1\right)\frac{\partial^2}{\partial\psi\partial\theta}f\left(\tilde{\psi},\theta\right)\left(\psi'-\psi''\right),\end{aligned}$$

where $\tilde{\psi}$ is some value between ψ' and ψ''. Since $\psi' - \psi''' \geq \psi' - \psi''$, we can conclude that Eq.(25) holds if we can show $-\frac{\partial^2}{\partial\psi\partial\theta}\pi_M^*\left(\hat{\psi},\theta\right) > \frac{\partial^2}{\partial\psi\partial\theta}f\left(\tilde{\psi},\theta\right)$. As we can verify, both $\frac{\partial^2}{\partial\psi\partial\theta}\pi_M^*\left(\psi,\theta\right)$ and

$\frac{\partial^2}{\partial\psi\partial\theta}f(\psi,\theta)$ are decreasing in ψ for $\forall\psi \in \left(0,\frac{1}{3}\right)$ when $\theta < 0.153$. Therefore,

$$\begin{aligned}
\frac{\partial^2}{\partial\psi\partial\theta}\pi_M^*\left(\hat{\psi},\theta\right) + \frac{\partial^2}{\partial\psi\partial\theta}f\left(\tilde{\psi},\theta\right) &< \frac{\partial^2}{\partial\psi\partial\theta}\pi_M^*(0,\theta) + \frac{\partial^2}{\partial\psi\partial\theta}f(0,\theta) \\
&= -\frac{2\left(2+13\theta-3\theta^2+\theta^3\right)}{27(1-\theta)^3} \\
&< 0.
\end{aligned}$$

Altogether, we have shown that Eq.(25) holds for $\forall\theta \in \left(0,\frac{1}{2}\right)$, which means the objective function is decreasing in θ.

Proof of Proposition 8. (i) Consider the locally envy-free equilibrium in which the $n-1$ niche firms' bids (labeled such that $b_N^n < b_N^{n-1} < ... < b_N^2$) are

$$\begin{cases}
b_N^n = \pi_N^{n-1(1)} - \pi_N^{n(1)} \\
b_N^i = \pi_N^{i-1(1)} - \pi_N^{i(1)} + b_N^{i+1}, \qquad\qquad i = 3, ..., n-1 \\
b_N^2 = \max\{\pi_N^{1(2)} - \pi_N^{2(1)}, 0\} + b_N^3
\end{cases}$$

and the mainstream firm bids any amount greater than b_N^2. As a result, M wins the first position and niche firms stay in the 2nd through the nth positions.

Now we investigate under what conditions the above bidding strategy profile is indeed an equilibrium. It is easy to see that all niche firms have no profitable deviations. To ensure no profitable deviation of M, $\pi_M^1 - b_N^2 \geq \pi_M^k - b_N^{k+1}$ has to be satisfied for all $k = 2, ..., n$ (Let $b_N^{n+1} = 0$). In other words, the following conditions have to be satisfied.

$$\pi_M^1 - \left(\max\{\pi_N^{1(2)}, \pi_N^{2(1)}\} - \pi_N^{n(1)}\right) \geq \pi_M^k - \left(\pi_N^{k(1)} - \pi_N^{n(1)}\right), \, k = 2, ..., n;$$

or equivalently,

$$\begin{cases}
\pi_M^1 - \pi_N^{1(2)} \geq \pi_M^k - \pi_N^{k(1)} \quad k = 2, ..., n \\
\pi_M^1 - \pi_N^{2(1)} \geq \pi_M^k - \pi_N^{k(1)} \quad k = 2, ..., n
\end{cases},$$

which are further equivalent to that

$$\begin{cases} \pi_M^*\left(\bar{\psi}_{-1},\theta\right)-\pi_M^*\left(\bar{\psi}_{-k},\theta\right)+\frac{1}{n-1}\left(1-\psi_k\right)f\left(\bar{\psi}_{-1},\theta\right)-\frac{1}{n-1}\left(1-\psi_1\right)f\left(\bar{\psi}_{-2},\theta\right)\geq 0 \\ \pi_M^*\left(\bar{\psi}_{-1},\theta\right)-\pi_M^*\left(\bar{\psi}_{-k},\theta\right)+\frac{1}{n-1}\left(1-\psi_k\right)f\left(\bar{\psi}_{-1},\theta\right)-\frac{1}{n-1}\left(1-\psi_2\right)f\left(\bar{\psi}_{-1},\theta\right)\geq 0 \end{cases}$$

holds for all $k=2,...,n$. By Lemma 2, the LHS of the first inequality is decreasing in θ. Also, it is easy to show that the LHS of the second inequality is decreasing in θ, given that $\frac{\partial}{\partial\theta}f\left(\psi,\theta\right)>0$. As a result, as long as the values of $\{\psi_i\}_{i=1}^n$ satisfy

$$\begin{cases} \pi_M^*\left(\bar{\psi}_{-1},0\right)-\pi_M^*\left(\bar{\psi}_{-k},0\right)+\frac{1}{n-1}\left(1-\psi_k\right)f\left(\bar{\psi}_{-1},0\right)-\frac{1}{n-1}\left(1-\psi_1\right)f\left(\bar{\psi}_{-2},0\right)>0 \\ \pi_M^*\left(\bar{\psi}_{-1},0\right)-\pi_M^*\left(\bar{\psi}_{-k},0\right)+\frac{1}{n-1}\left(1-\psi_k\right)f\left(\bar{\psi}_{-1},0\right)-\frac{1}{n-1}\left(1-\psi_2\right)f\left(\bar{\psi}_{-1},0\right)>0 \end{cases}$$

for $k=2,...,n$, which is a loose parametric condition that can be satisfied under most values of $\{\psi_i\}_{i=1}^n$, we can conclude that there exists $\hat{\theta}\left(\psi_1,...,\psi_n\right)$ such that when $0<\theta<\hat{\theta}\left(\psi_1,...,\psi_n\right)$, the described bidding strategy profile is an equilibrium.

(ii) Similarly, consider the locally envy-free equilibrium in which the $n-1$ niche firms' bids (labeled such that $b_N^{n-1}<b_N^{n-2}<...<b_N^1$) are

$$\begin{cases} b_N^{n-1}=\pi_N^{n-2(n)}-\pi_N^{n-1(n)}+b_M \\ b_N^i=\pi_N^{i-1(n)}-\pi_N^{i(n)}+b_N^{i+1}, \qquad i=2,...,n-2 \\ b_N^1>b_N^2 \end{cases}$$

and the mainstream firm bids $b_M=\pi_M^{n-1}-\pi_M^n$. As a result, the niche firms stay in the first $n-1$ positions and M stays in the last one.

To see under what conditions the above bidding strategy profile is indeed an equilibrium, we need to ensure that any niche firm does not have profitable deviation, such that $\pi_N^{n(n-1)}\leq\pi_N^{k(n)}-b_N^{k+1}=\pi_N^{n-1(n)}-\left(\pi_M^{n-1}-\pi_M^n\right)$ $(k=1,...,n-1)$, as well as that M does not have profitable deviation, such that $\pi_M^k-b_N^k\leq\pi_M^n$ for $k=2,...,n-1$. (Note that no deviation to the first position for M can easily hold as long as b_N^1 is high enough.) We can organize these conditions as

$$\begin{cases} \pi_M^{n-1} - \pi_M^n + \pi_N^{n(n-1)} - \pi_N^{n-1(n)} \leq 0 \\ \pi_M^k - \pi_M^{n-1} + \pi_N^{n-1(n)} - \pi_N^{k-1(n)} \leq 0, \quad k = 2, ..., n-1; \end{cases}$$

or equivalently,

$$\begin{cases} \pi_M^* \left(\bar{\psi}_{-(n-1)}, \theta\right) - \pi_M^* \left(\bar{\psi}_{-n}, \theta\right) + \frac{1}{n-1} \left(1 - \psi_n\right) f \left(\bar{\psi}_{-(n-1)}, \theta\right) - \frac{1}{n-1} \left(1 - \psi_{n-1}\right) f \left(\bar{\psi}_{-n}, \theta\right) \leq 0 \\ \pi_M^* \left(\bar{\psi}_{-k}, \theta\right) - \pi_M^* \left(\bar{\psi}_{-(n-1)}, \theta\right) + \frac{1}{n-1} \left(1 - \psi_{n-1}\right) f \left(\bar{\psi}_{-n}, \theta\right) - \frac{1}{n-1} \left(1 - \psi_{k-1}\right) f \left(\bar{\psi}_{-n}, \theta\right) \leq 0, \end{cases}$$

where $k = 2, ..., n-1$. By Lemma 2, the LHS of the first inequality is decreasing in θ. Also, it is easy to show that the LHS of the second inequality is decreasing in θ, given that $\frac{\partial}{\partial \theta} f(\psi, \theta) > 0$. As a result, as long as the values of $\{\psi_i\}_{i=1}^n$ satisfy

$$\begin{cases} \pi_M^* \left(\bar{\psi}_{-(n-1)}, \frac{1}{2}\right) - \pi_M^* \left(\bar{\psi}_{-n}, \frac{1}{2}\right) + \frac{1}{n-1} \left(1 - \psi_n\right) f \left(\bar{\psi}_{-(n-1)}, \frac{1}{2}\right) - \frac{1}{n-1} \left(1 - \psi_{n-1}\right) f \left(\bar{\psi}_{-n}, \frac{1}{2}\right) < 0 \\ \pi_M^* \left(\bar{\psi}_{-k}, \frac{1}{2}\right) - \pi_M^* \left(\bar{\psi}_{-(n-1)}, \frac{1}{2}\right) + \frac{1}{n-1} \left(1 - \psi_{n-1}\right) f \left(\bar{\psi}_{-n}, \frac{1}{2}\right) - \frac{1}{n-1} \left(1 - \psi_{k-1}\right) f \left(\bar{\psi}_{-n}, \frac{1}{2}\right) < 0, \end{cases}$$

for $k = 2, ..., n-1$, which is not difficult to be satisfied under most values of $\{\psi_i\}_{i=1}^n$, we can conclude that there exists $\tilde{\theta}(\psi_1, ..., \psi_n)$ such that when $\tilde{\theta}(\psi_1, ..., \psi_n) < \theta < \frac{1}{2}$, the described bidding strategy profile is an equilibrium.